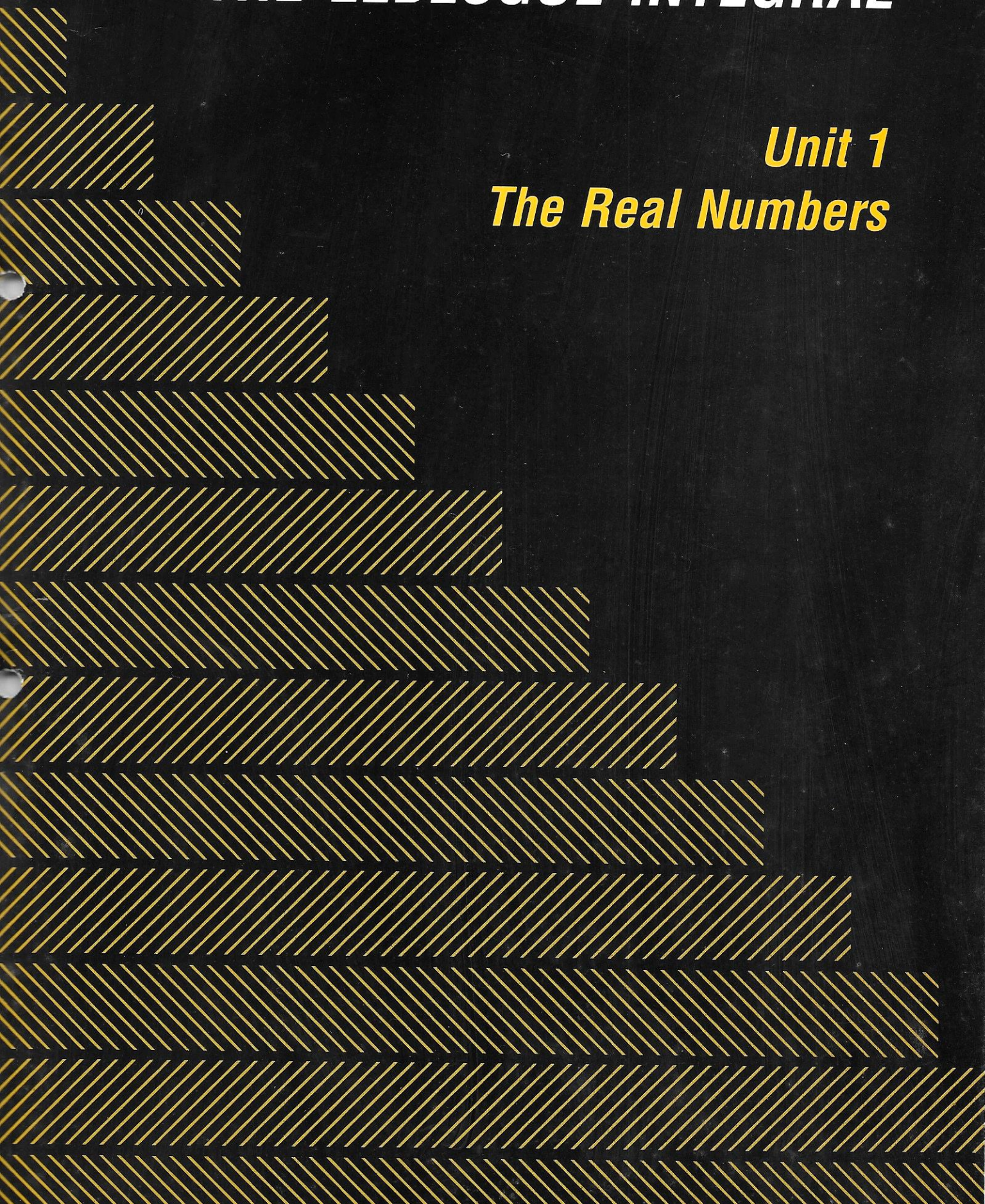


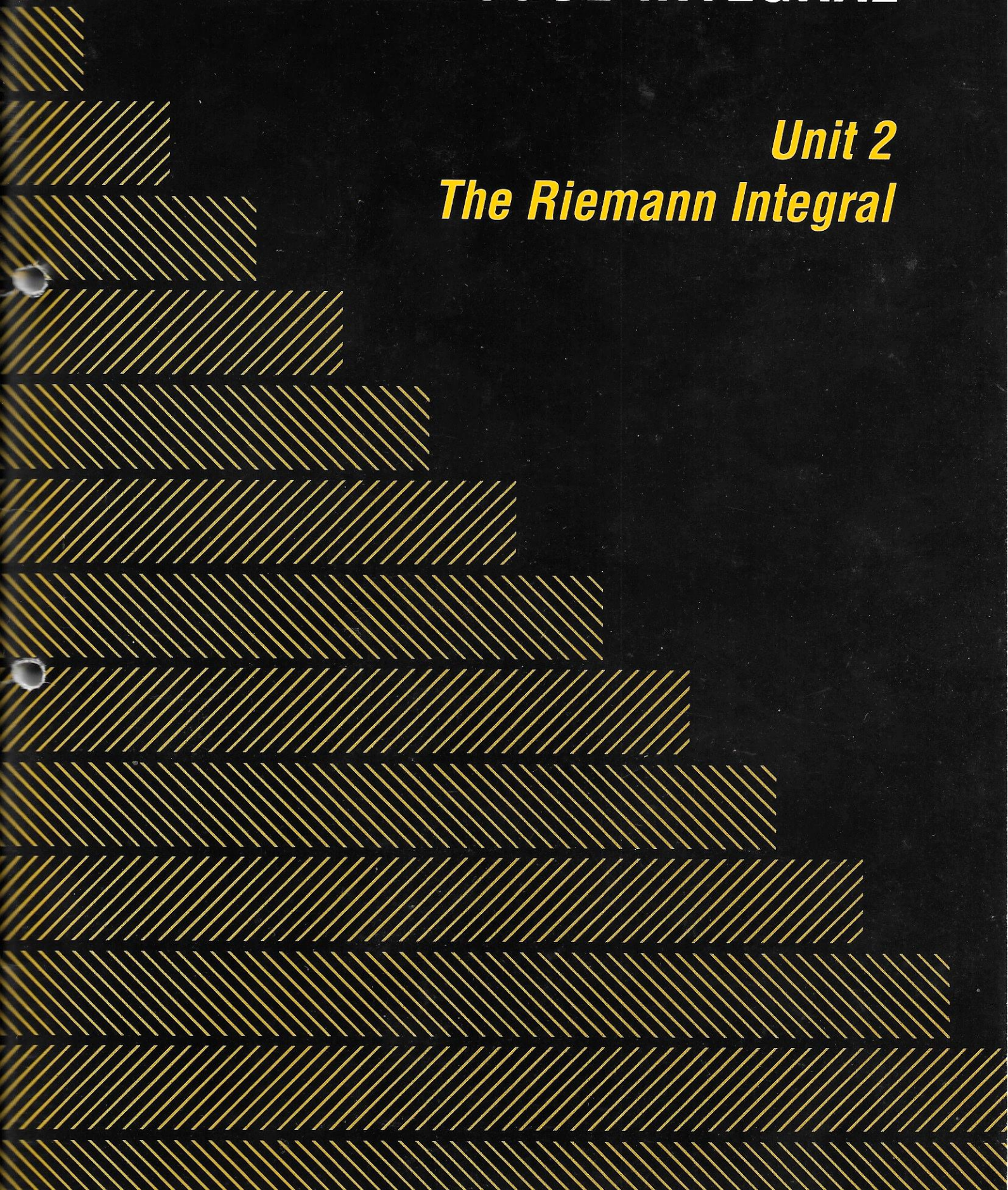
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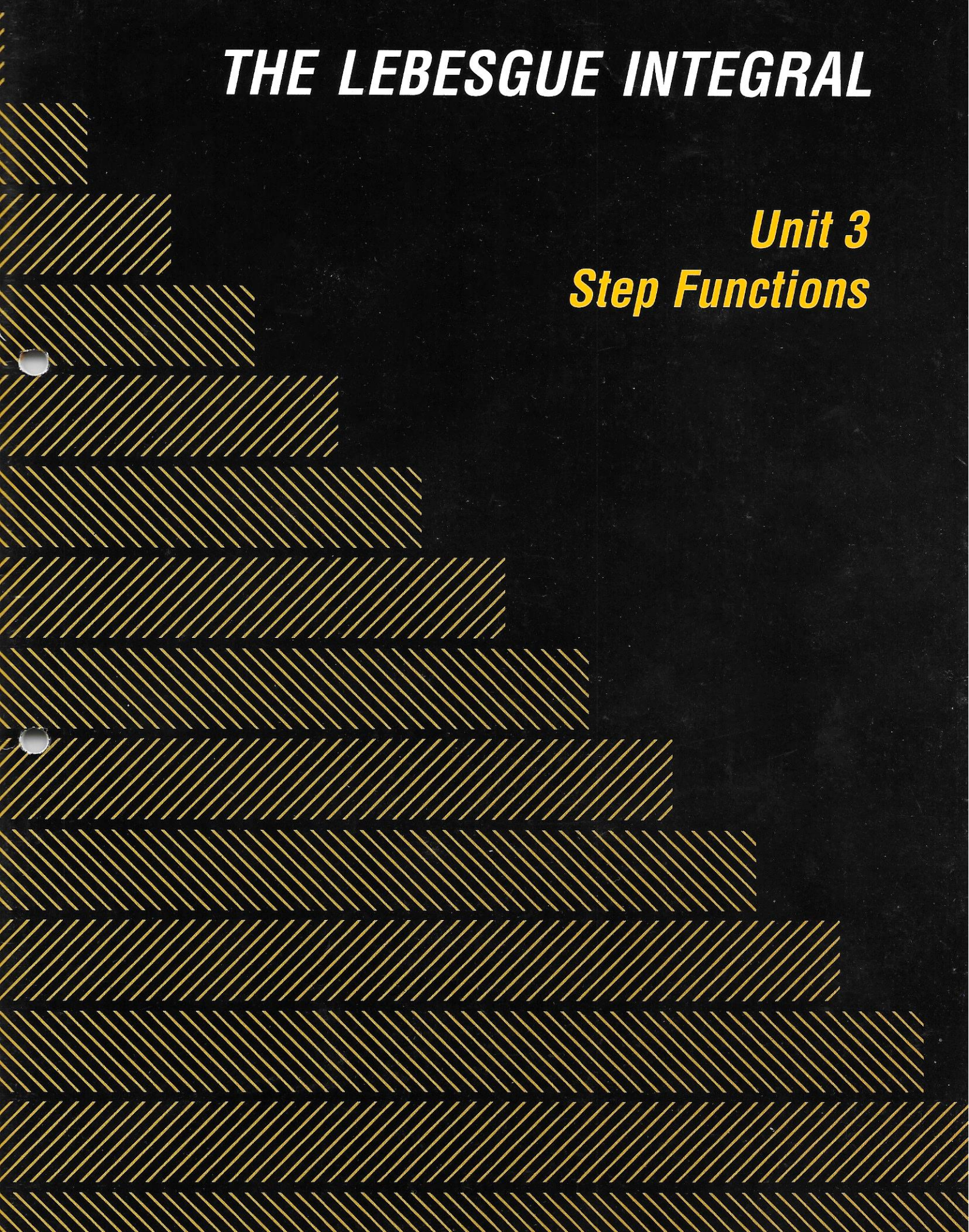
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Unit 5 ***Definite and*** ***Indefinite Integrals***

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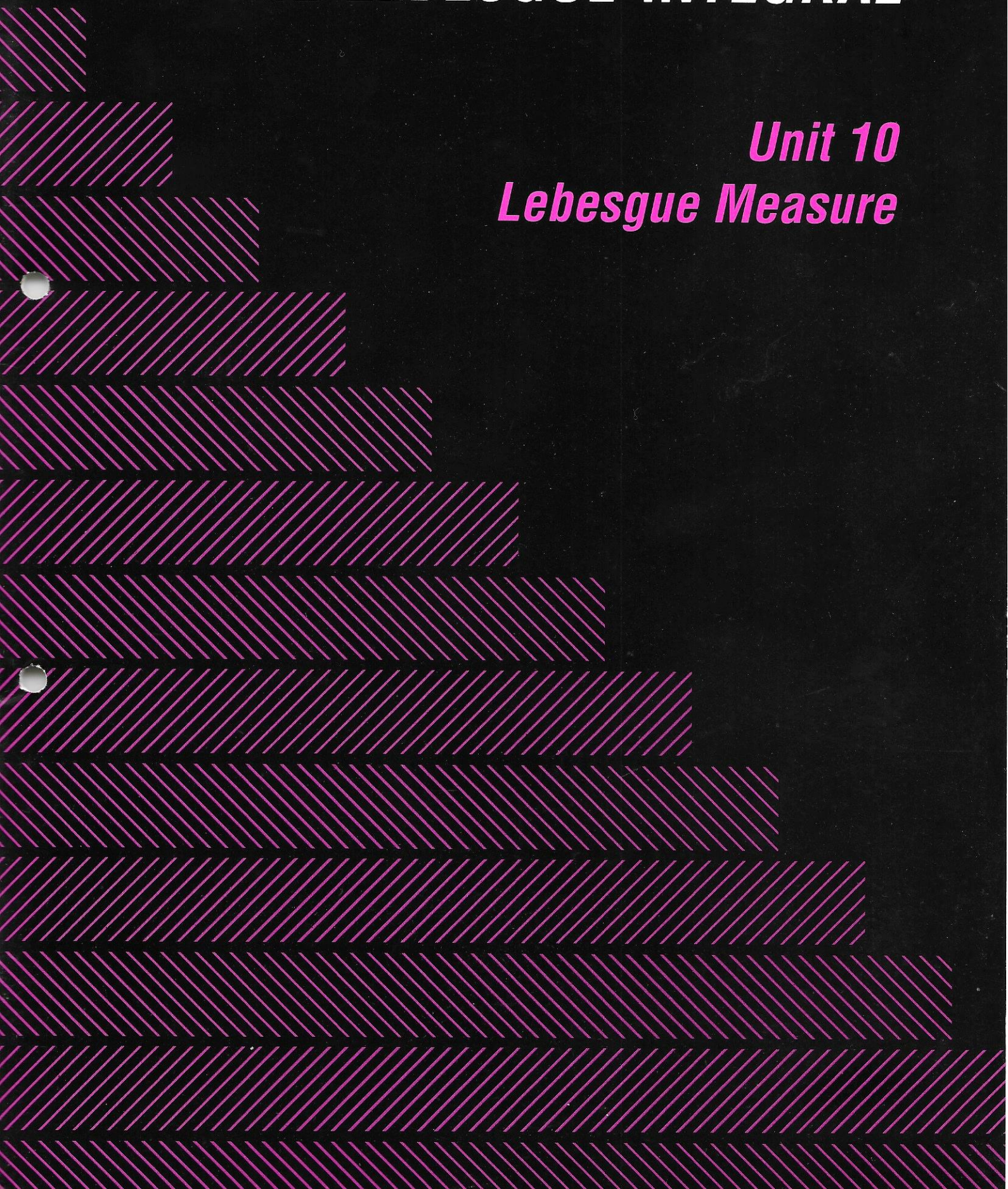
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LEBESGUE

INTEGRATION

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LEBESGUE INTEGRATION AND MEASURE

ALAN J. WEIR

*Reader in Mathematics and Education
University of Sussex*

Lebesgue integration is a technique of great power and elegance which can be applied in situations where other methods of integration fail. It is now one of the standard tools of modern mathematics, and forms part of many undergraduate courses in pure mathematics.

Dr Weir's book is aimed at the student who is meeting the Lebesgue integral for the first time. Defining the integral in terms of step functions provides an immediate link to elementary integration theory as taught in calculus courses. The more abstract concept of Lebesgue measure, which generalises the primitive notions of length, area and volume, is deduced later.

The explanations are simple and detailed with particular stress on motivation. Over 250 exercises accompany the text and are grouped at the ends of the sections to which they relate; notes on the solutions are given.

'The book is easy to read, partly because of the treatment adopted, and partly because of the quality of the exposition. Dr Weir's style is clear, friendly and informal; he shows how the results fit in with the reader's intuition; he highlights the important things and warns of the difficult things (these warnings when a hard bit is coming up are most confidence-preserving). He does not aim at maximum generality at the cost of understanding. The examples are chosen with care, many of them being, in effect, lemmas that will be needed later in the proofs of theorems.'

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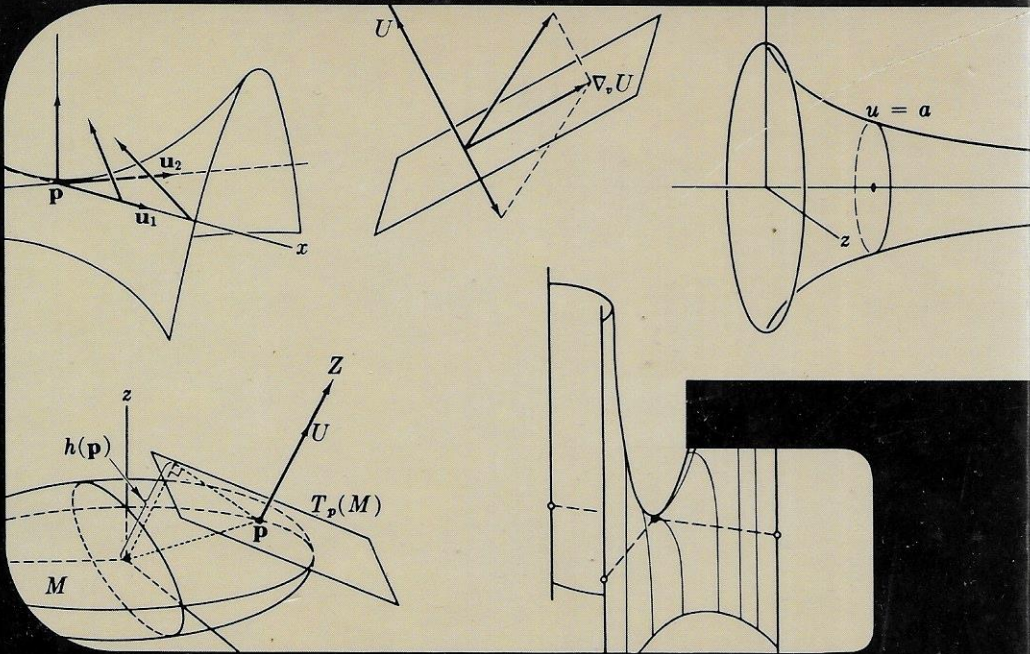
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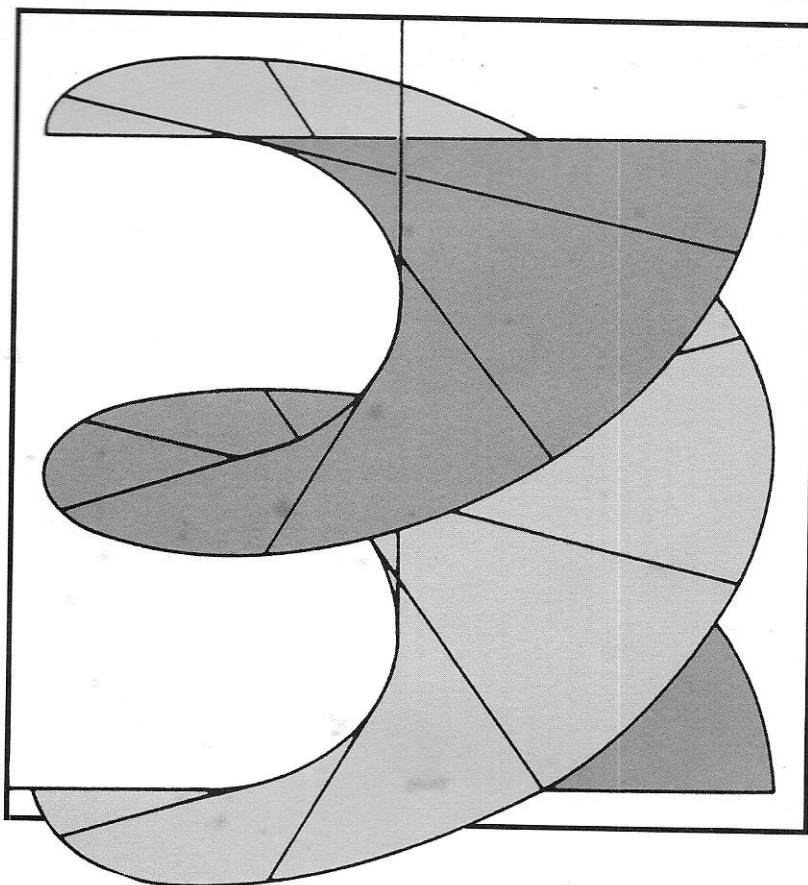
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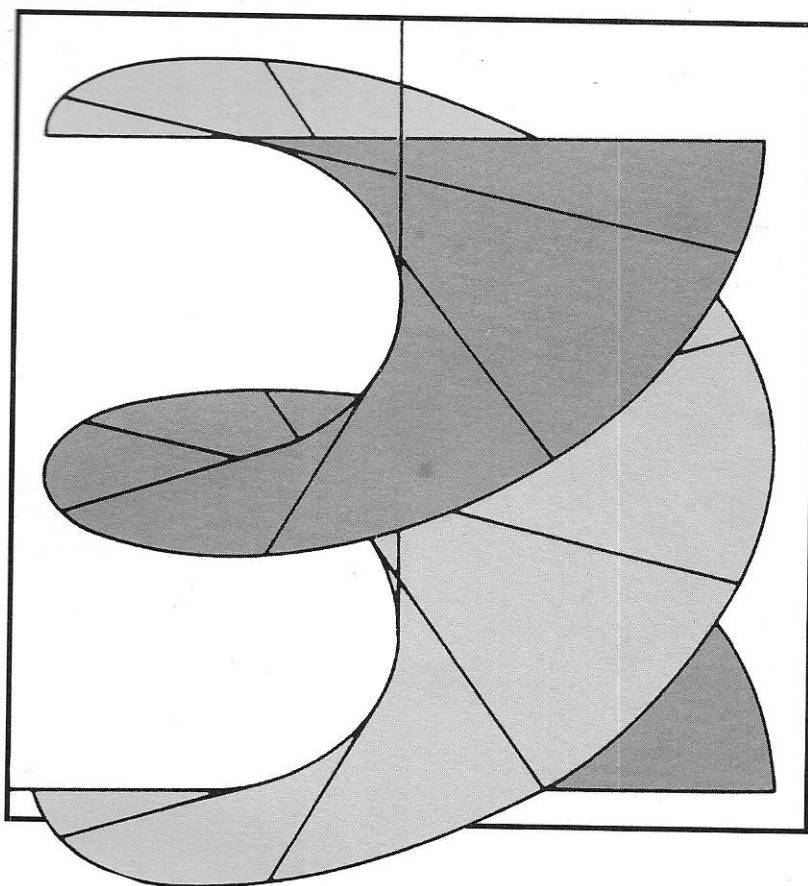


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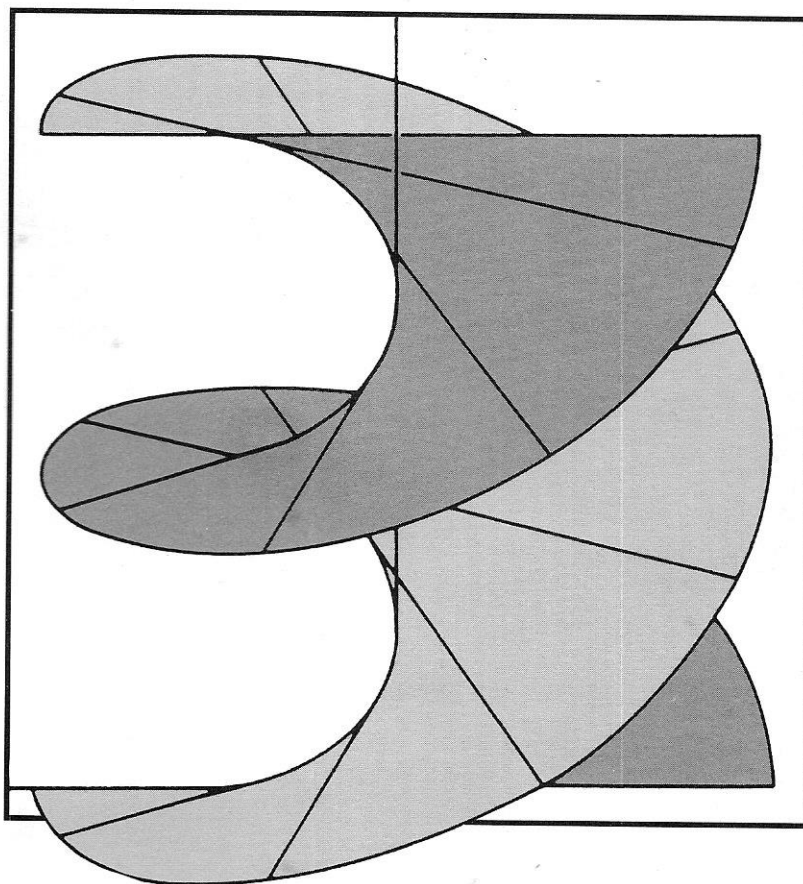


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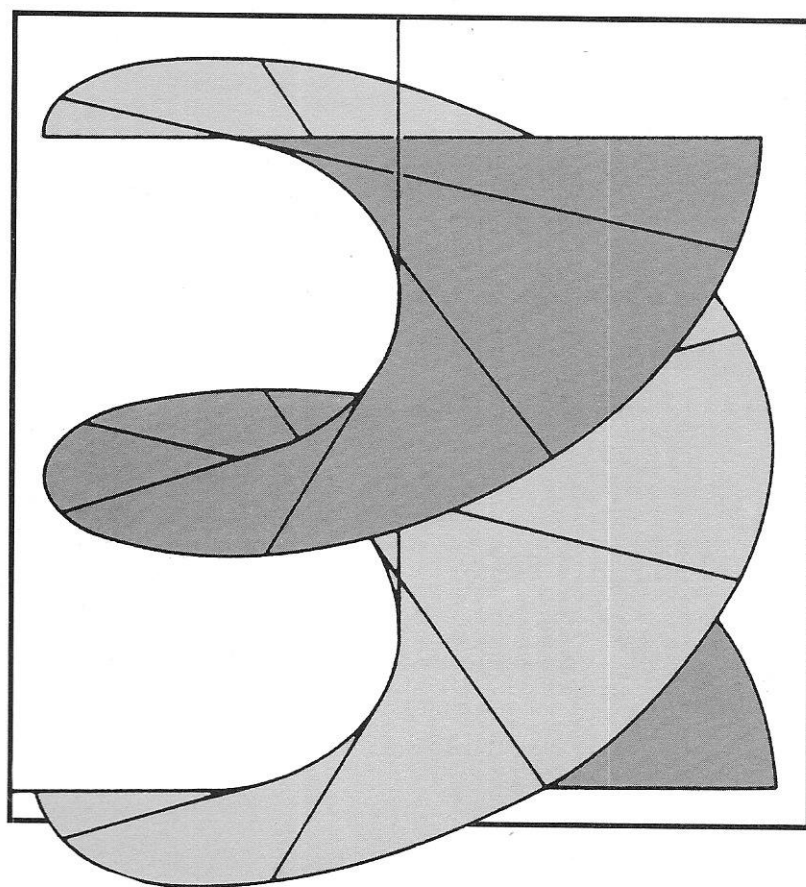
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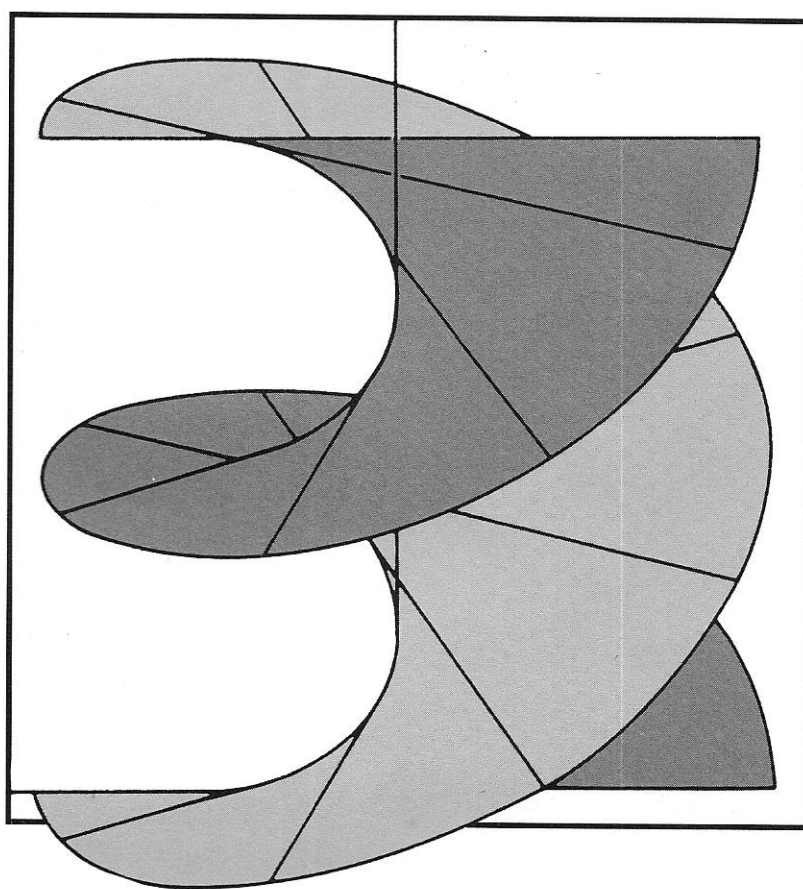


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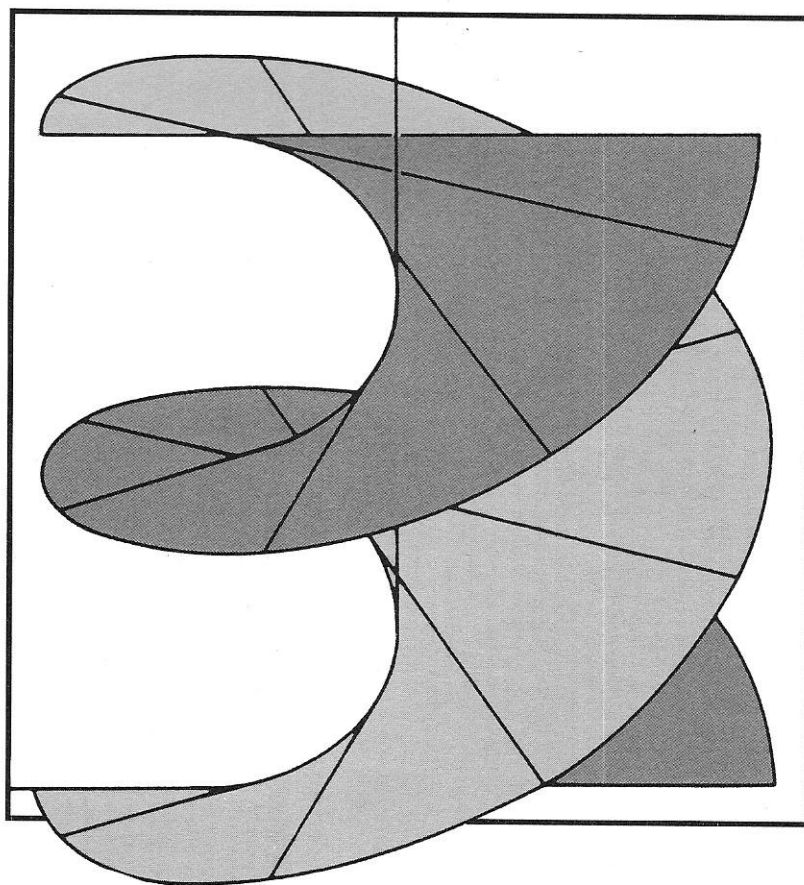


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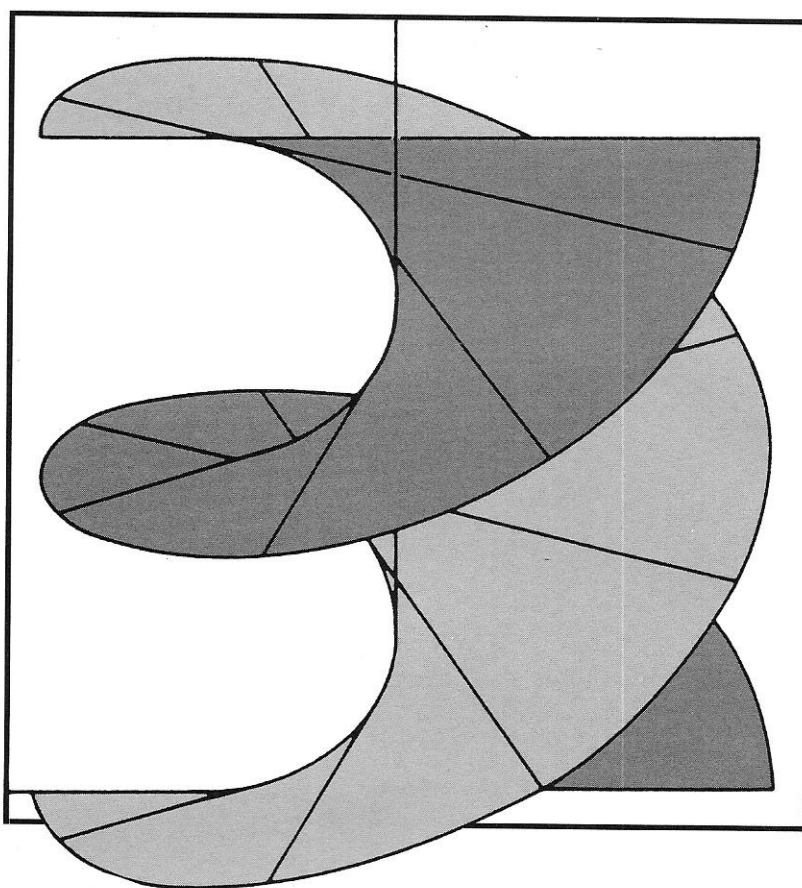


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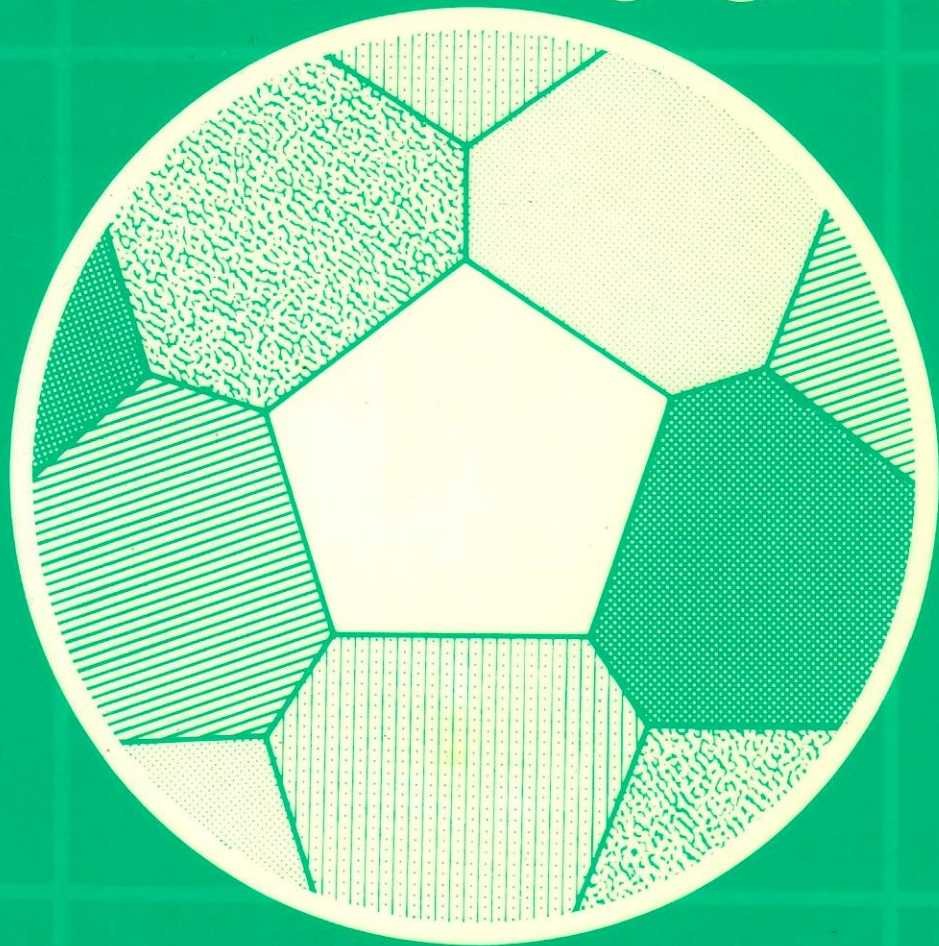
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A COMBINATORIAL
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DONALD W. BLACKETT

This book has been written as a text for a course in topology for undergraduates. It assumes a basic knowledge of calculus. The book treats selected topics from one, two, and three dimensions, primarily by combinatorial and algebraic methods.

After a chapter of examples, combinatorial methods are used to classify surfaces and to study covering surfaces. The concept of winding number is used to examine mappings into the sphere and vector fields on the plane and sphere. The fundamental theorem of algebra and the two-dimensional cases of the Brouwer fixed point theorem and the Borsuk-Ulam theorem are all covered. The homology of networks is developed and applied to the Kirchhoff-Maxwell laws and to a transportation problem. The final chapter provides a brief introduction to three-dimensional manifolds.

The concepts developed in each chapter are illustrated both by drawings and diagrams and by worked examples in the text. At the end of each chapter there is a set of exercises designed to reinforce students' understanding of the material covered and to develop certain topics further.

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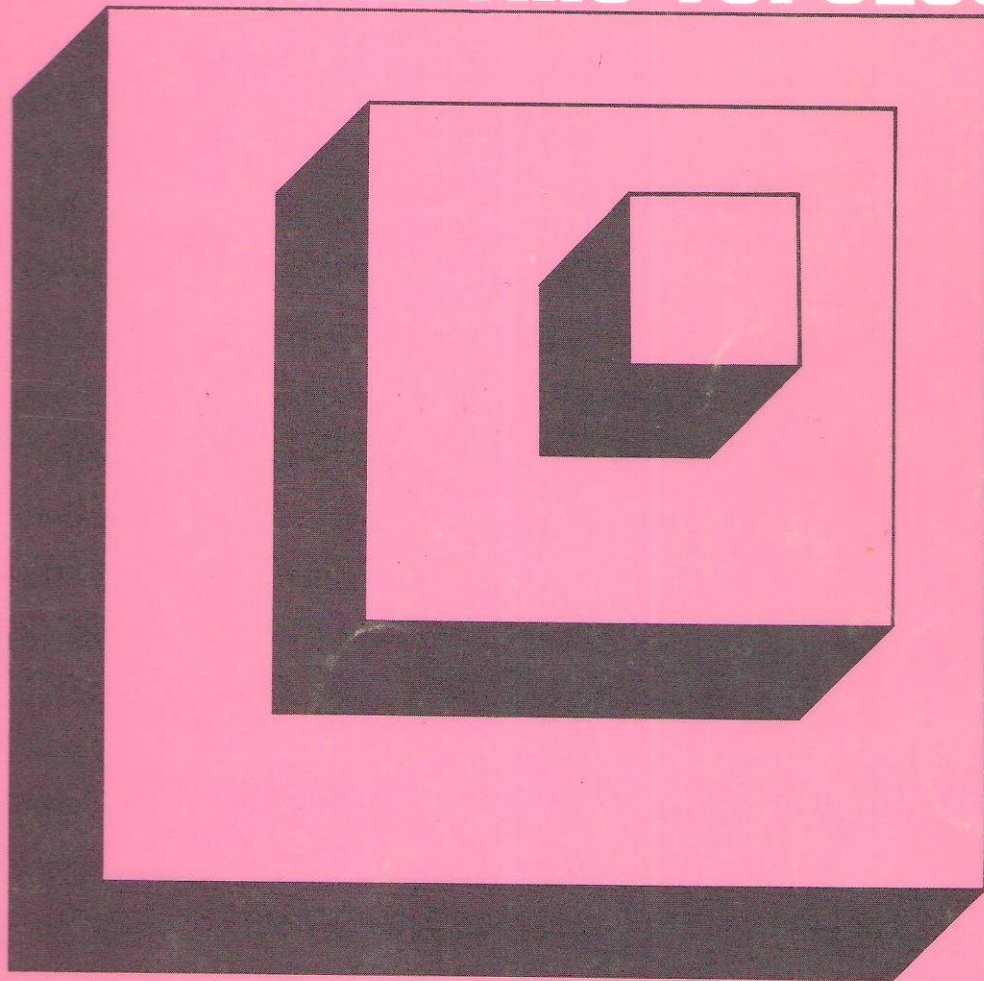
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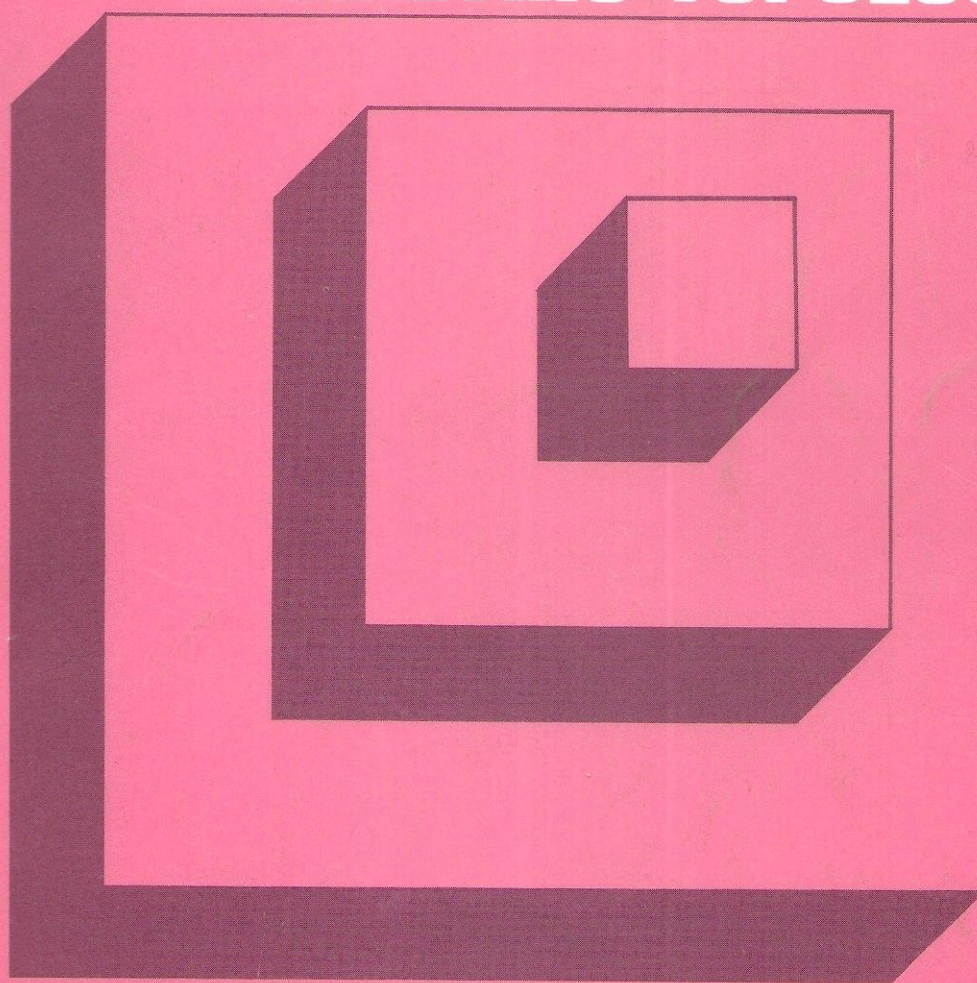
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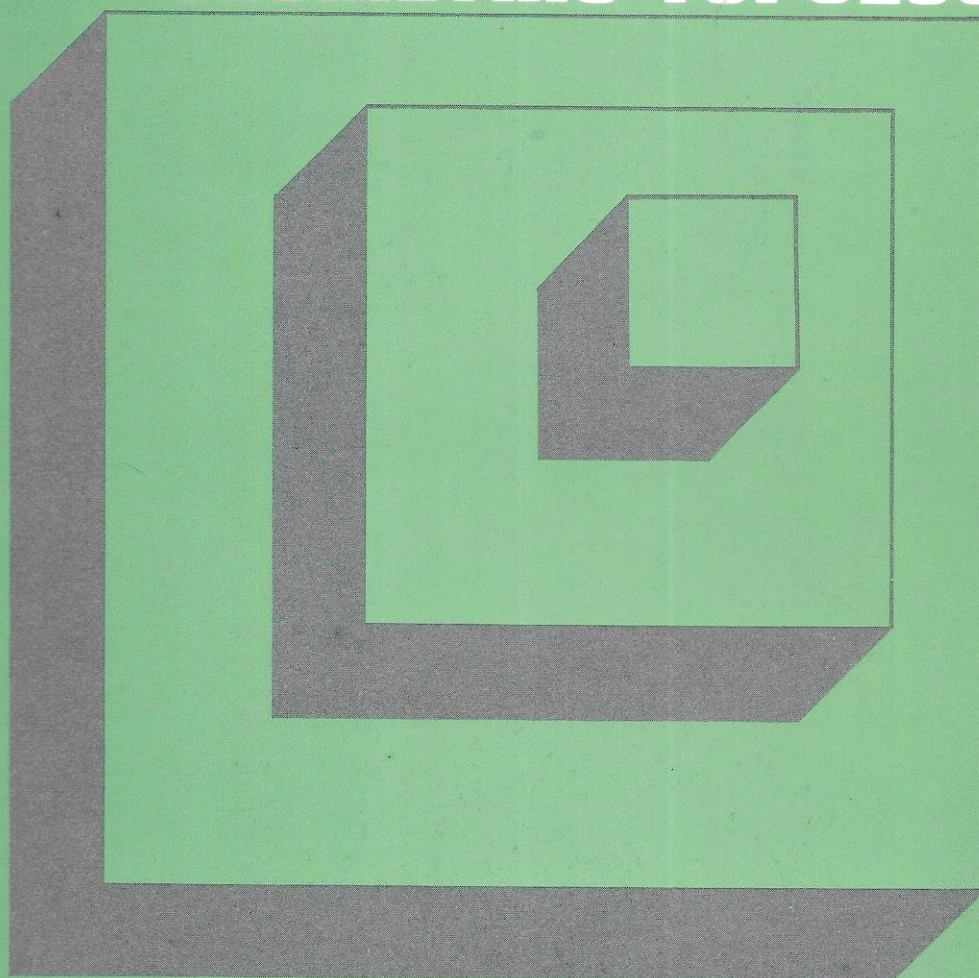
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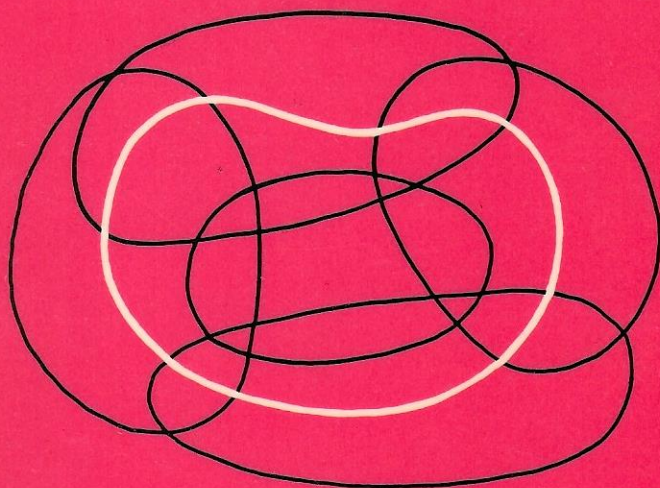
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The author is a Fellow of New College and Lecturer in Mathematics in the University of Oxford. He has previously taught at the Massachusetts Institute of Technology and at Manchester University.

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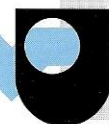
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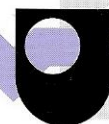


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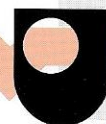
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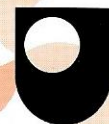


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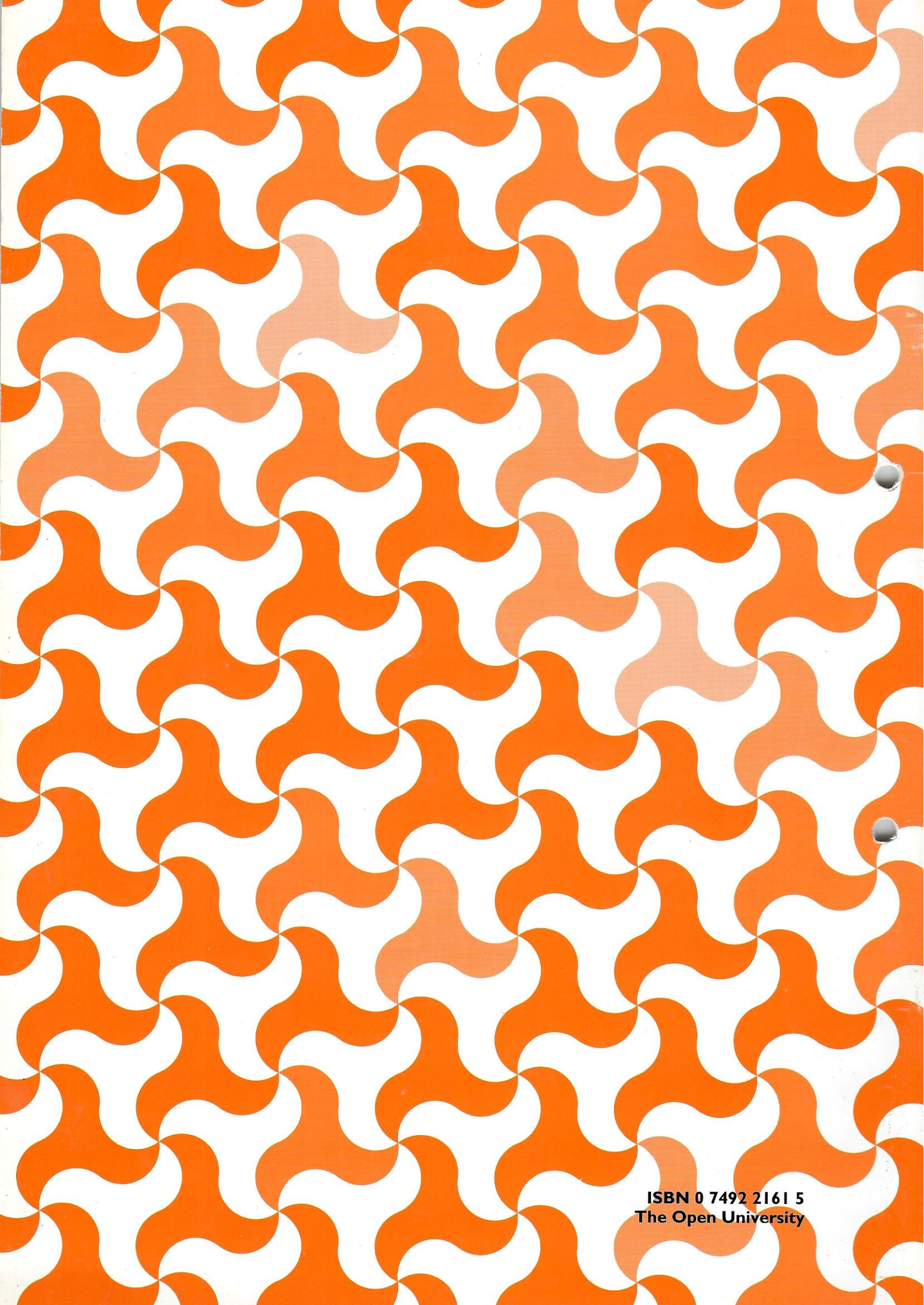
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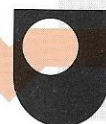
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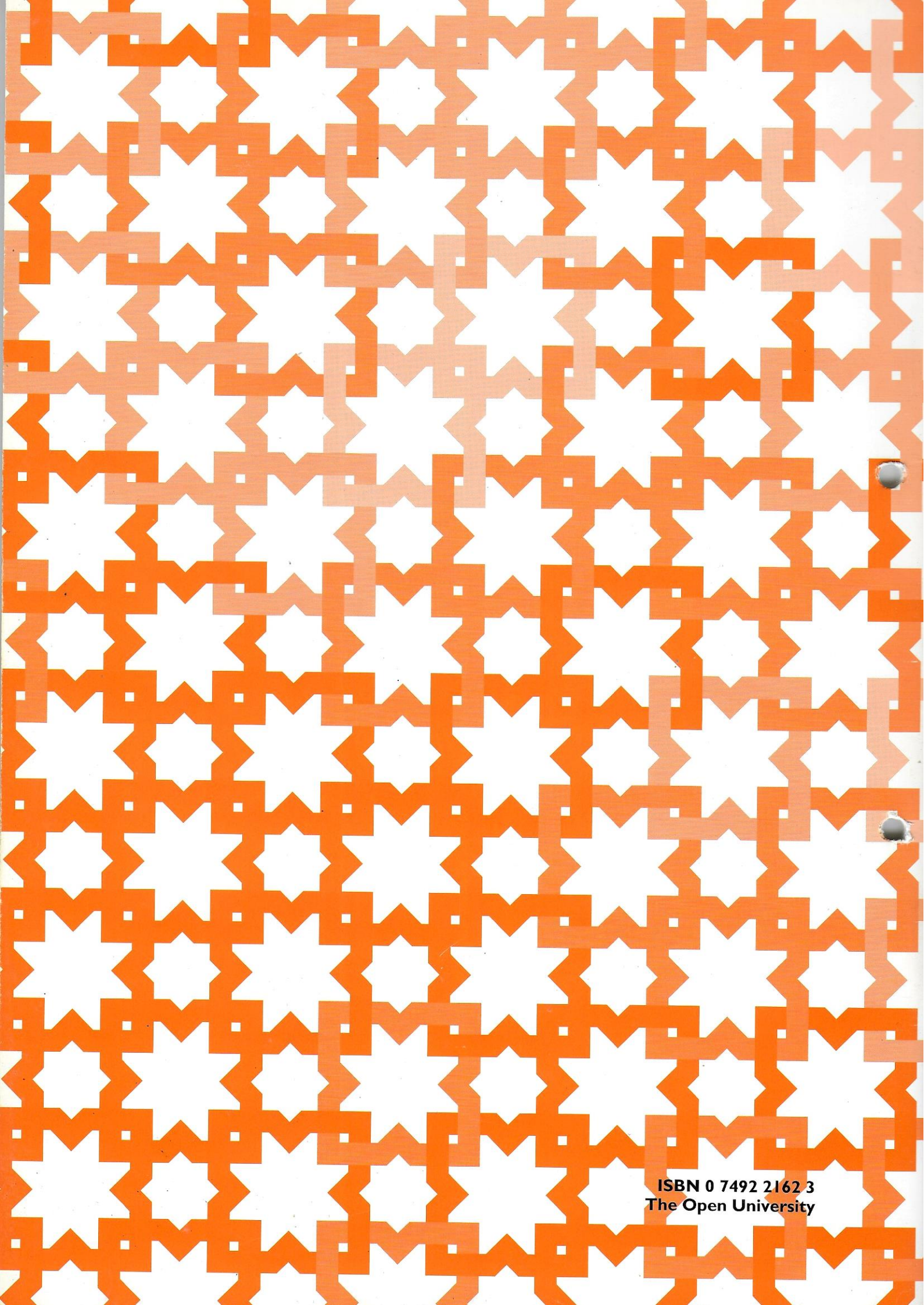
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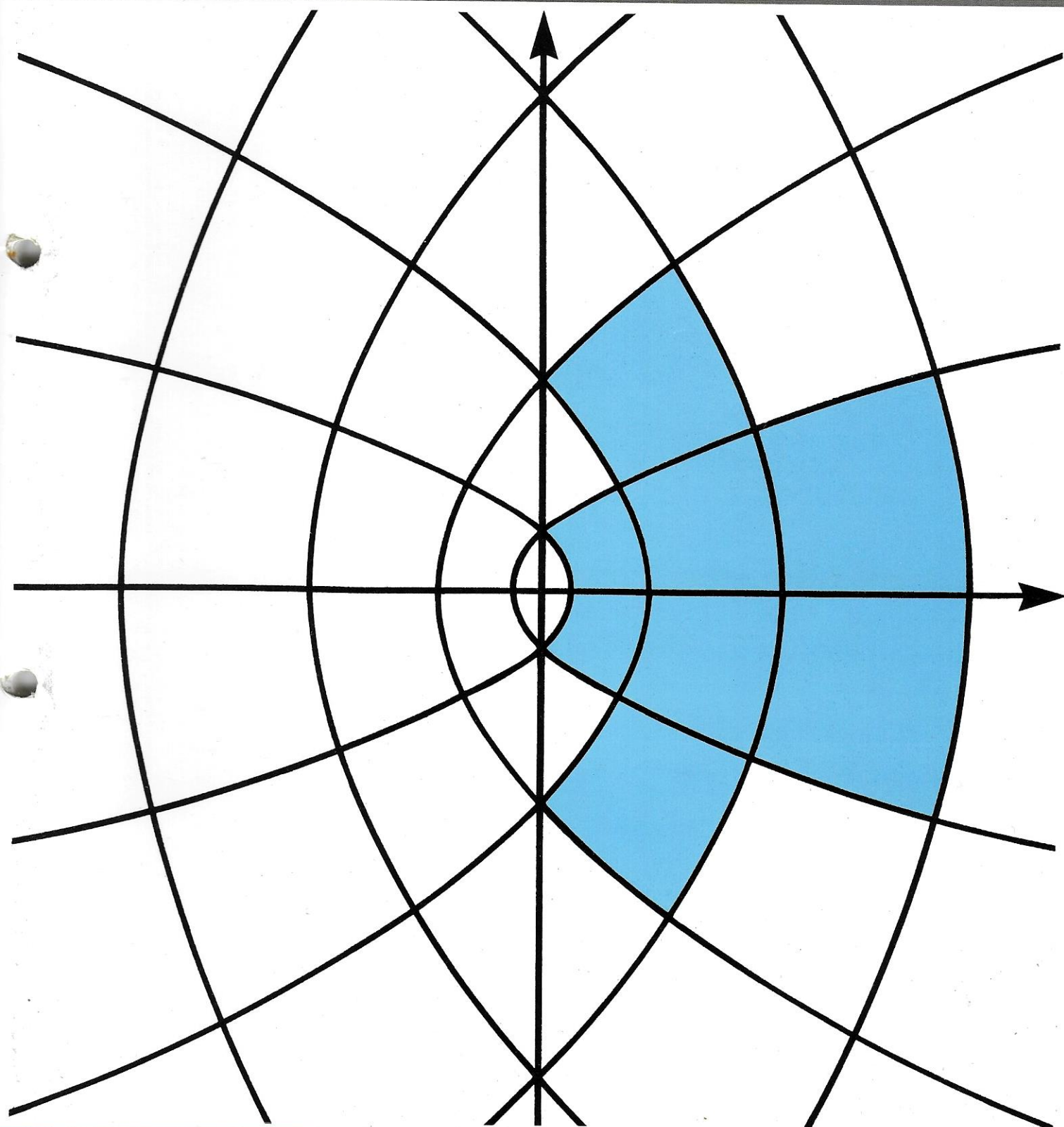
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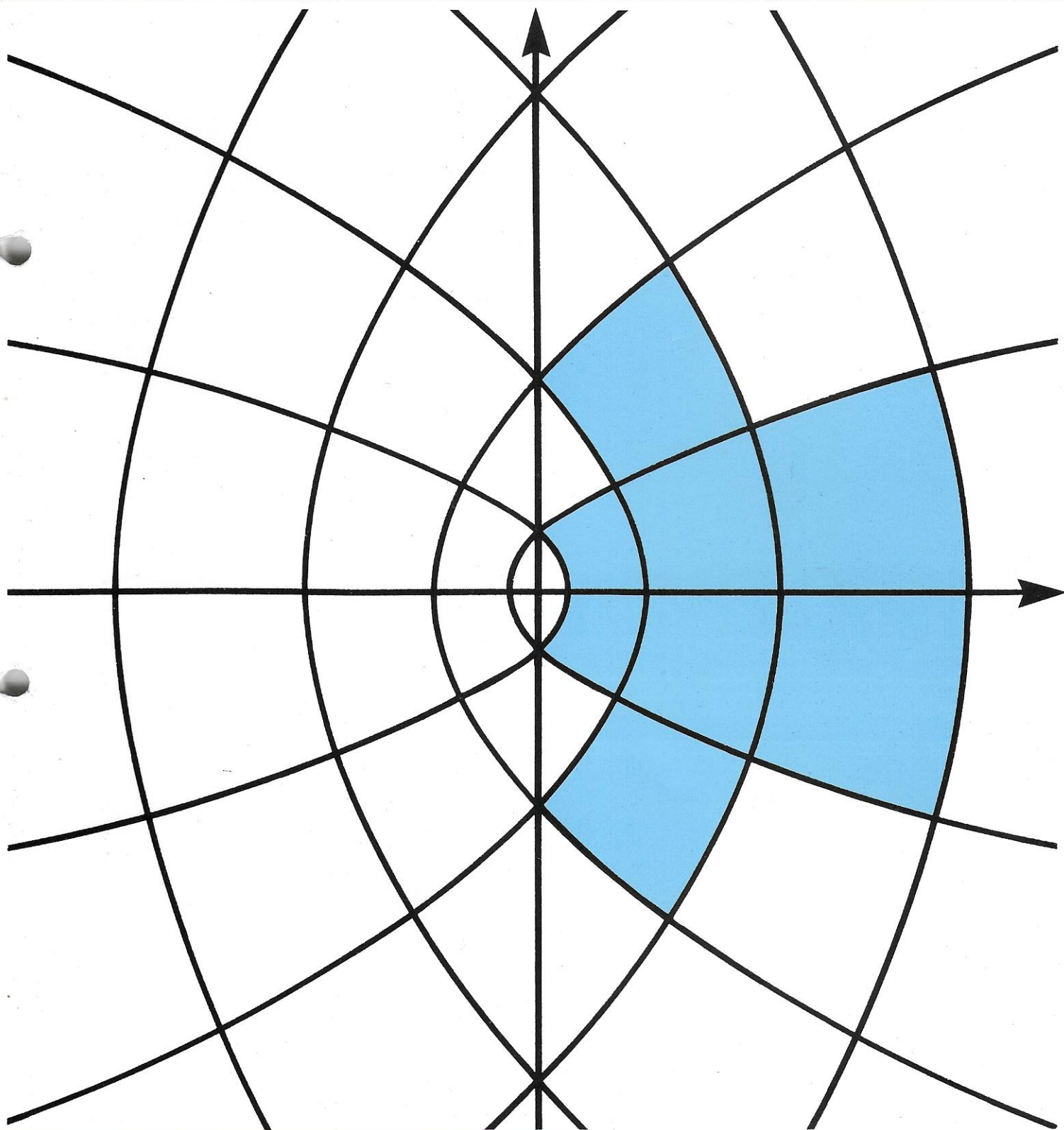
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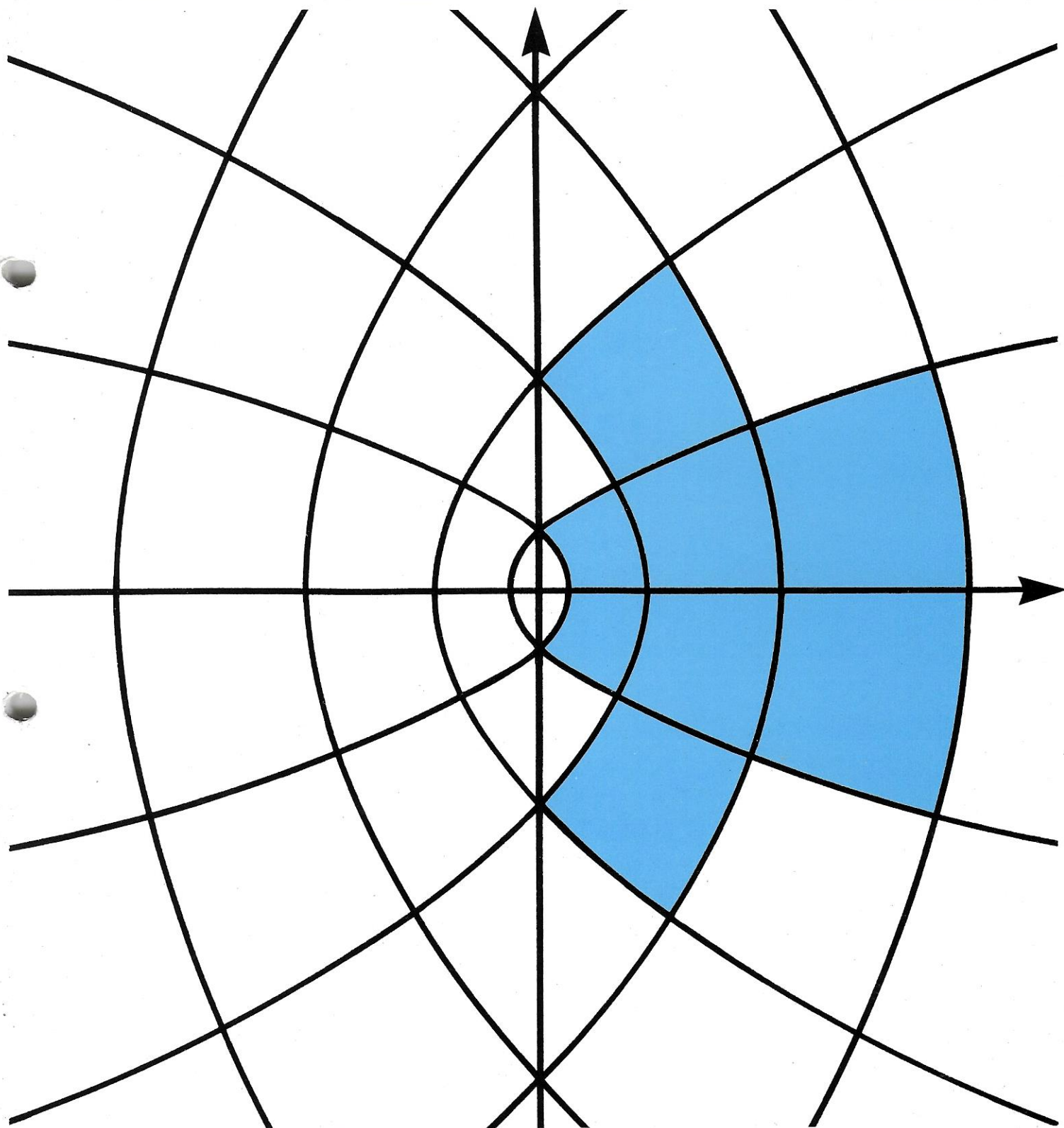
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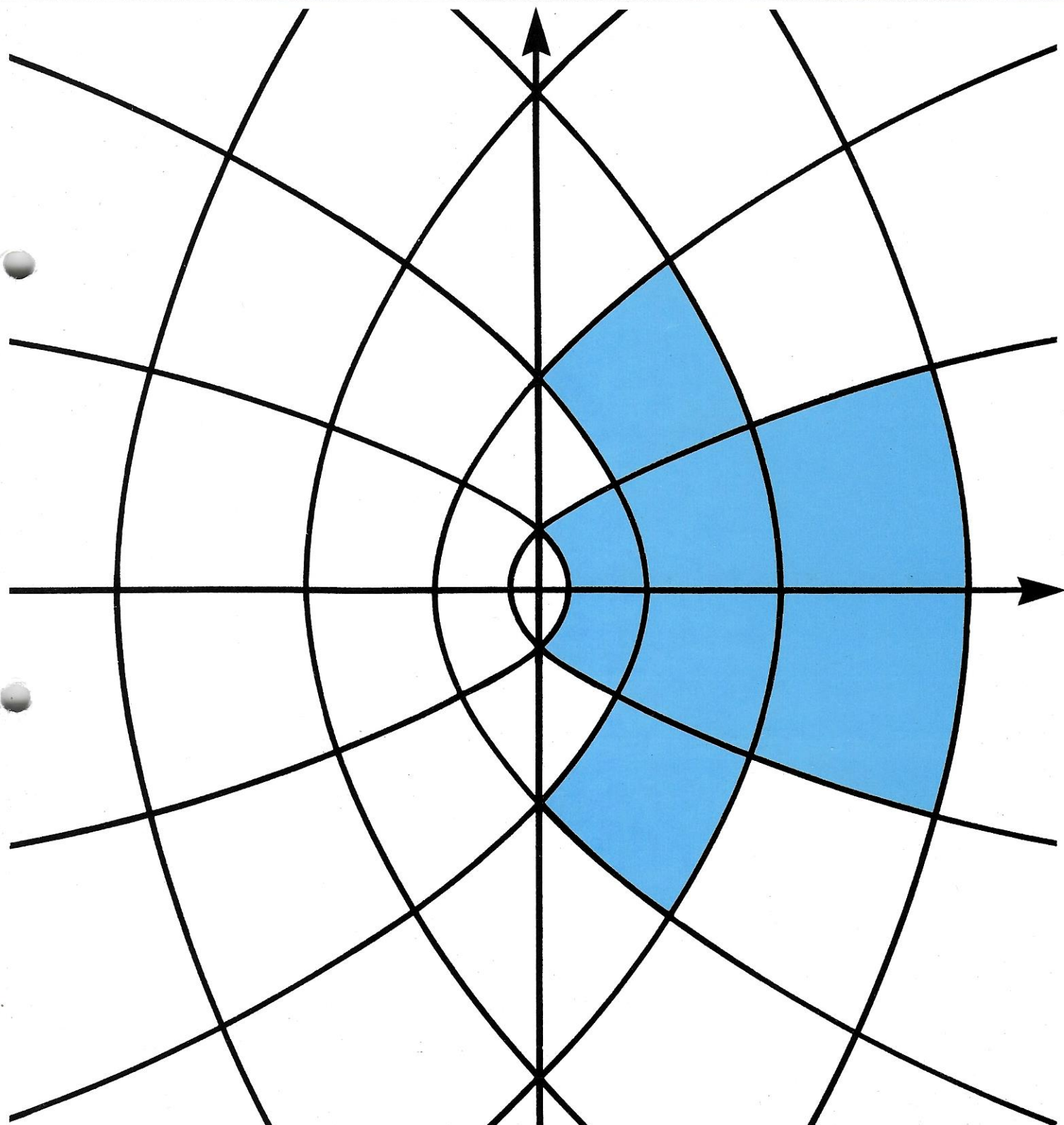
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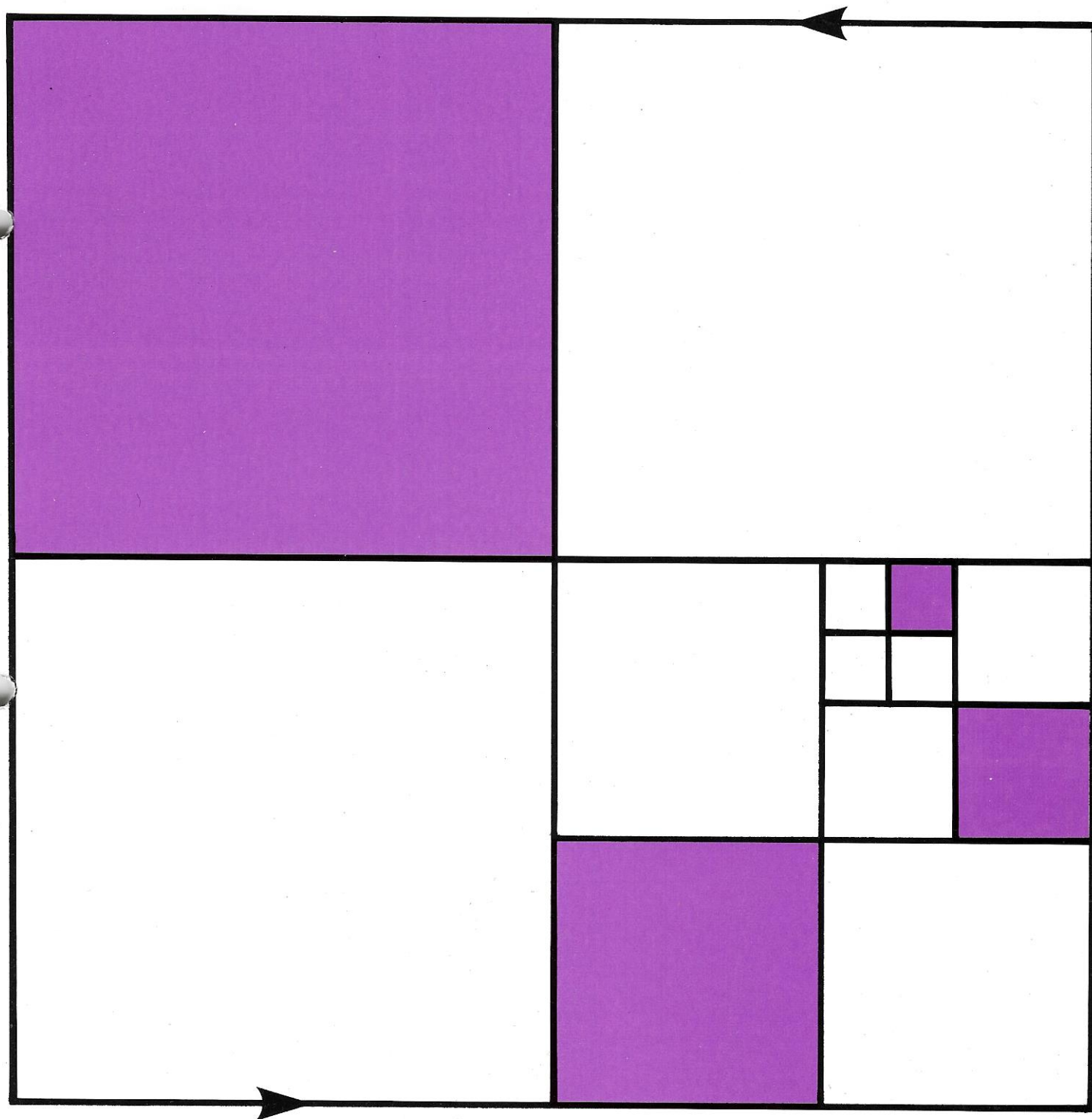
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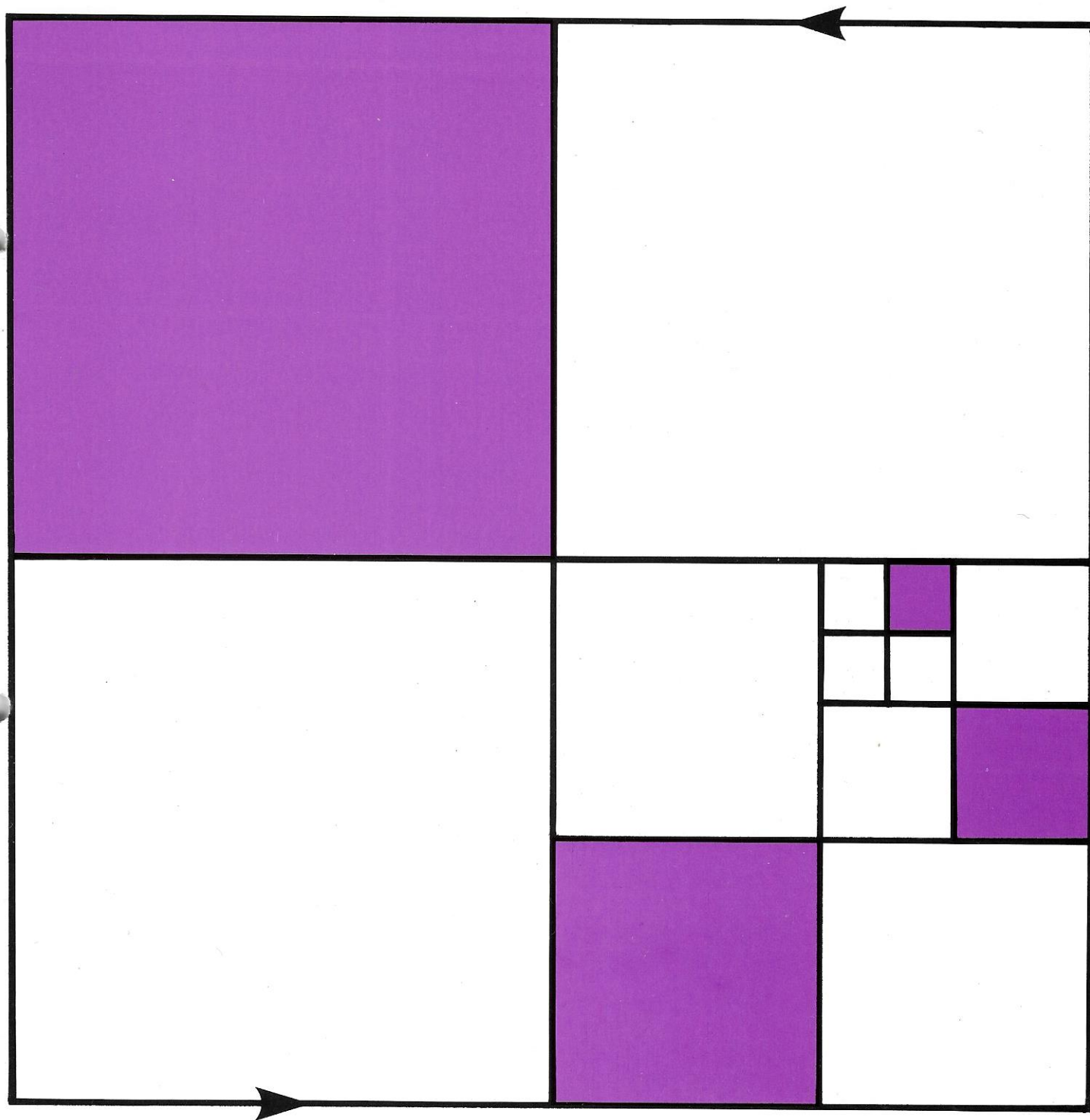
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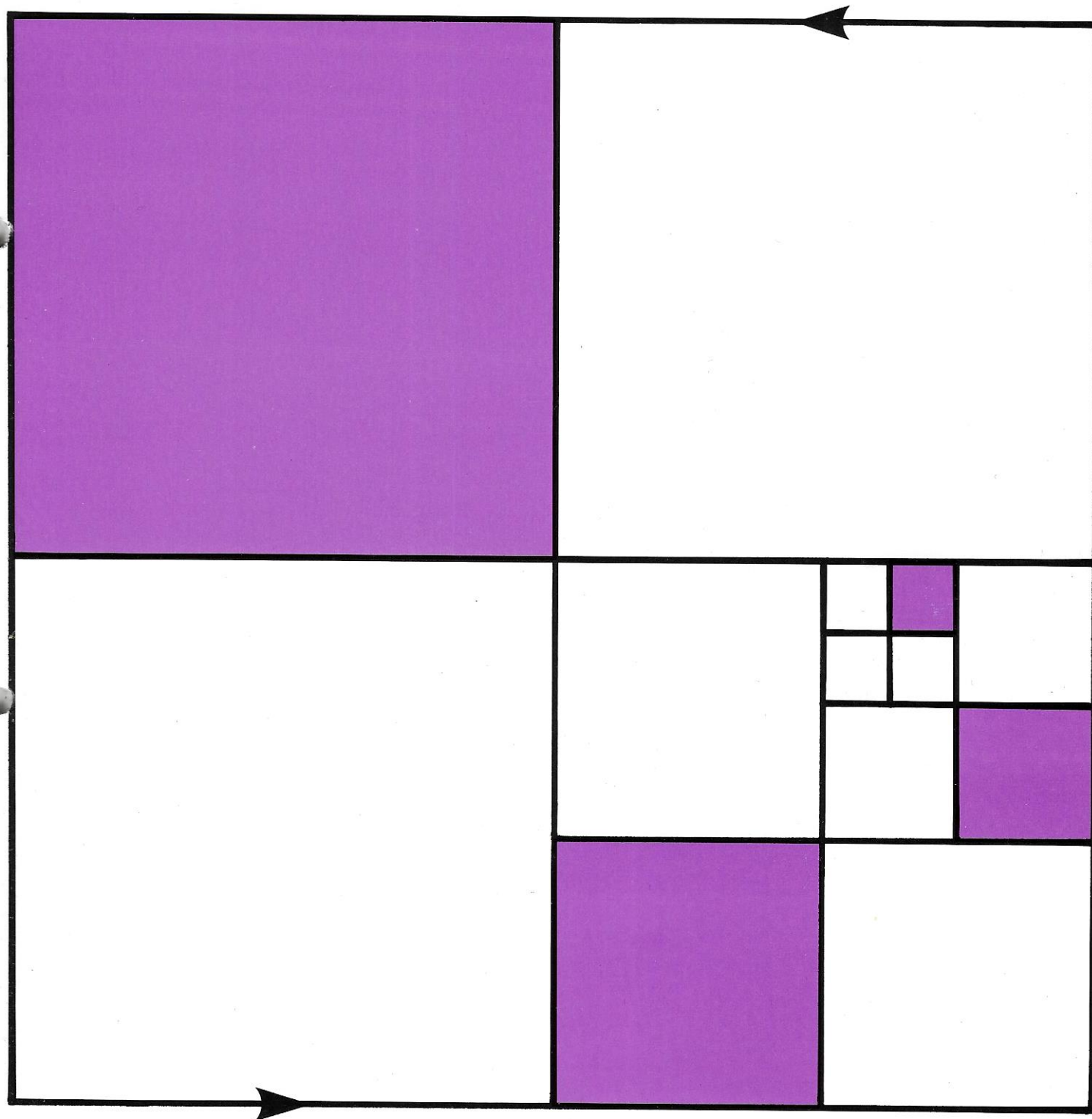
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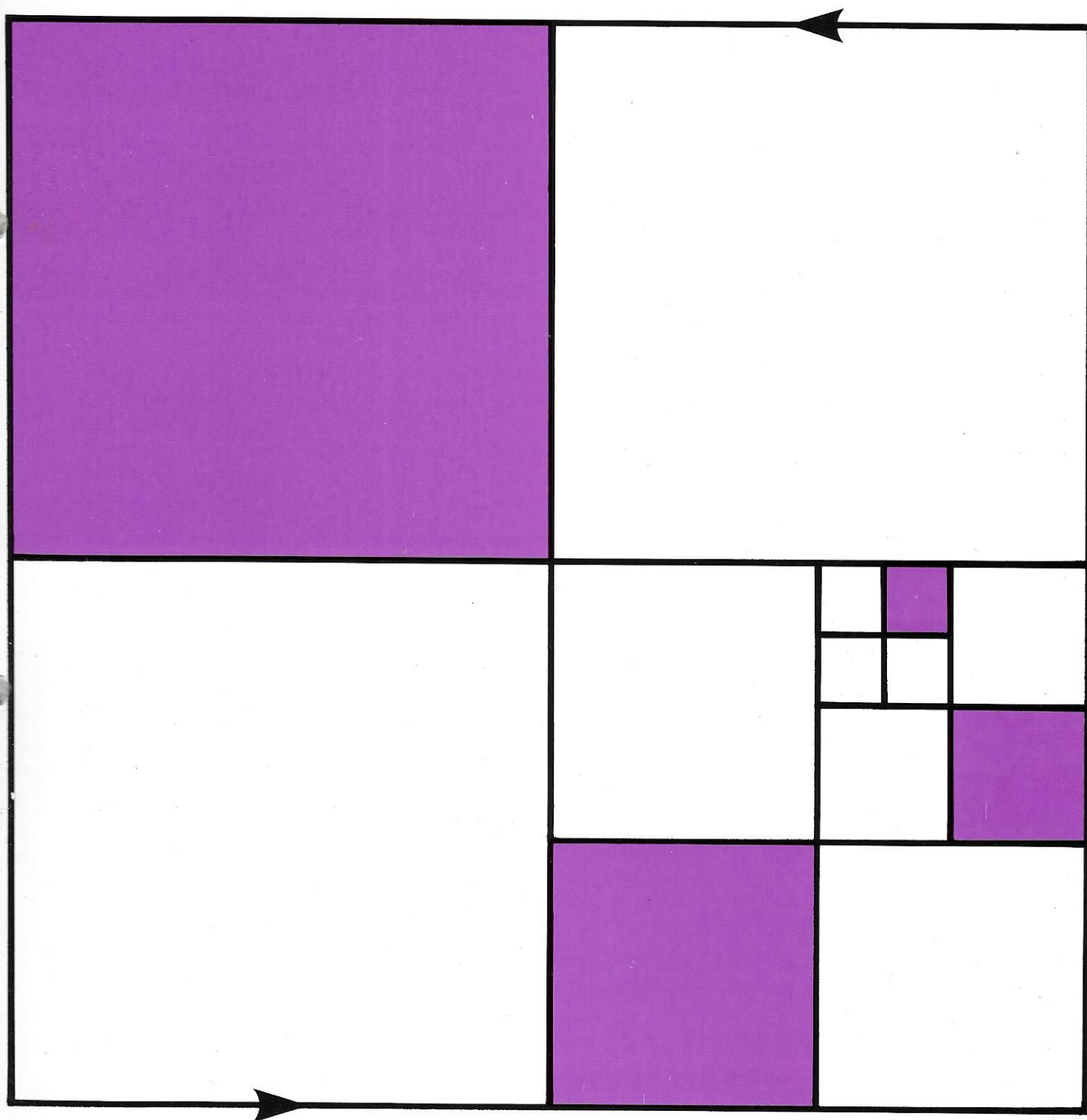
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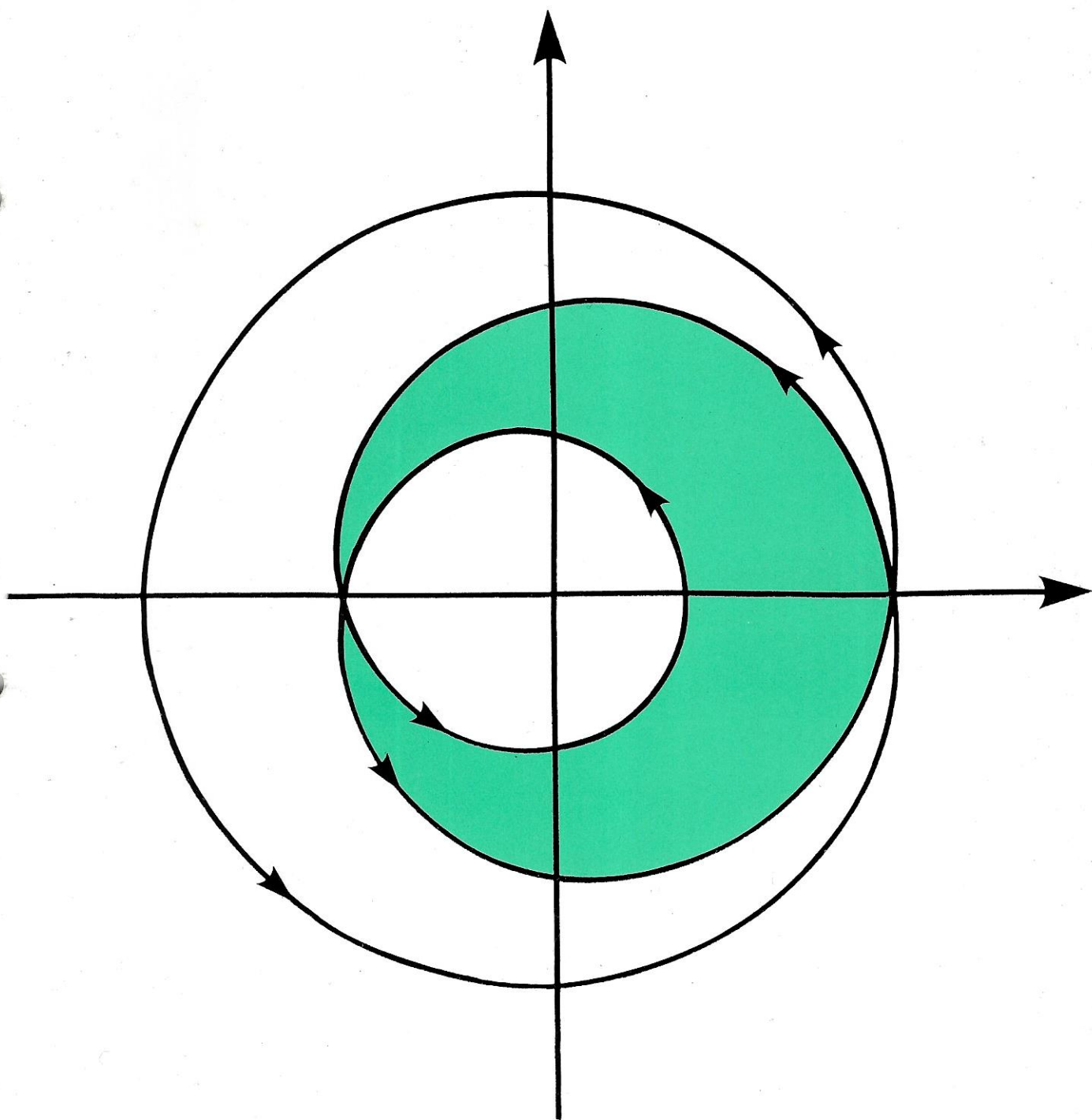
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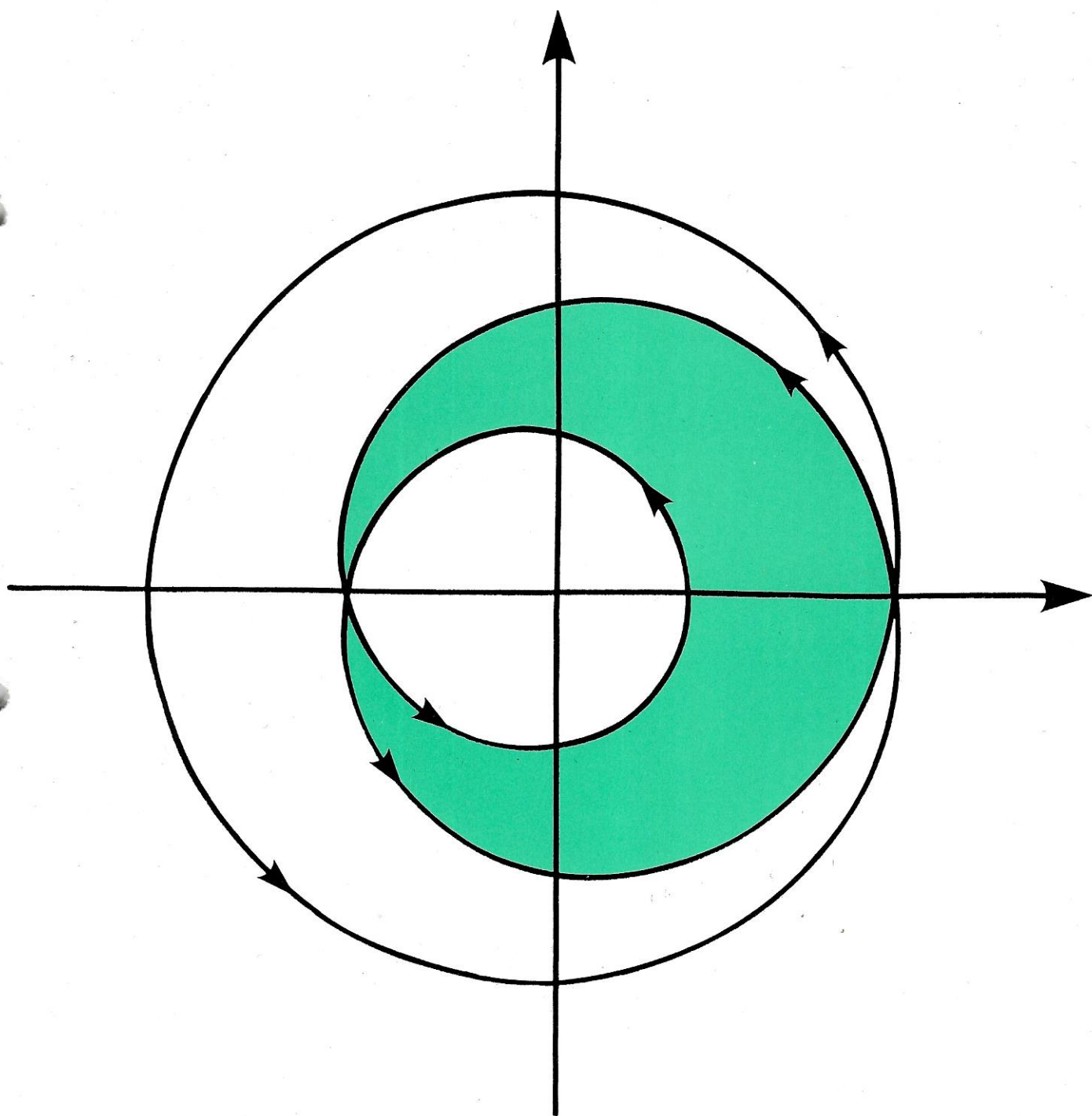
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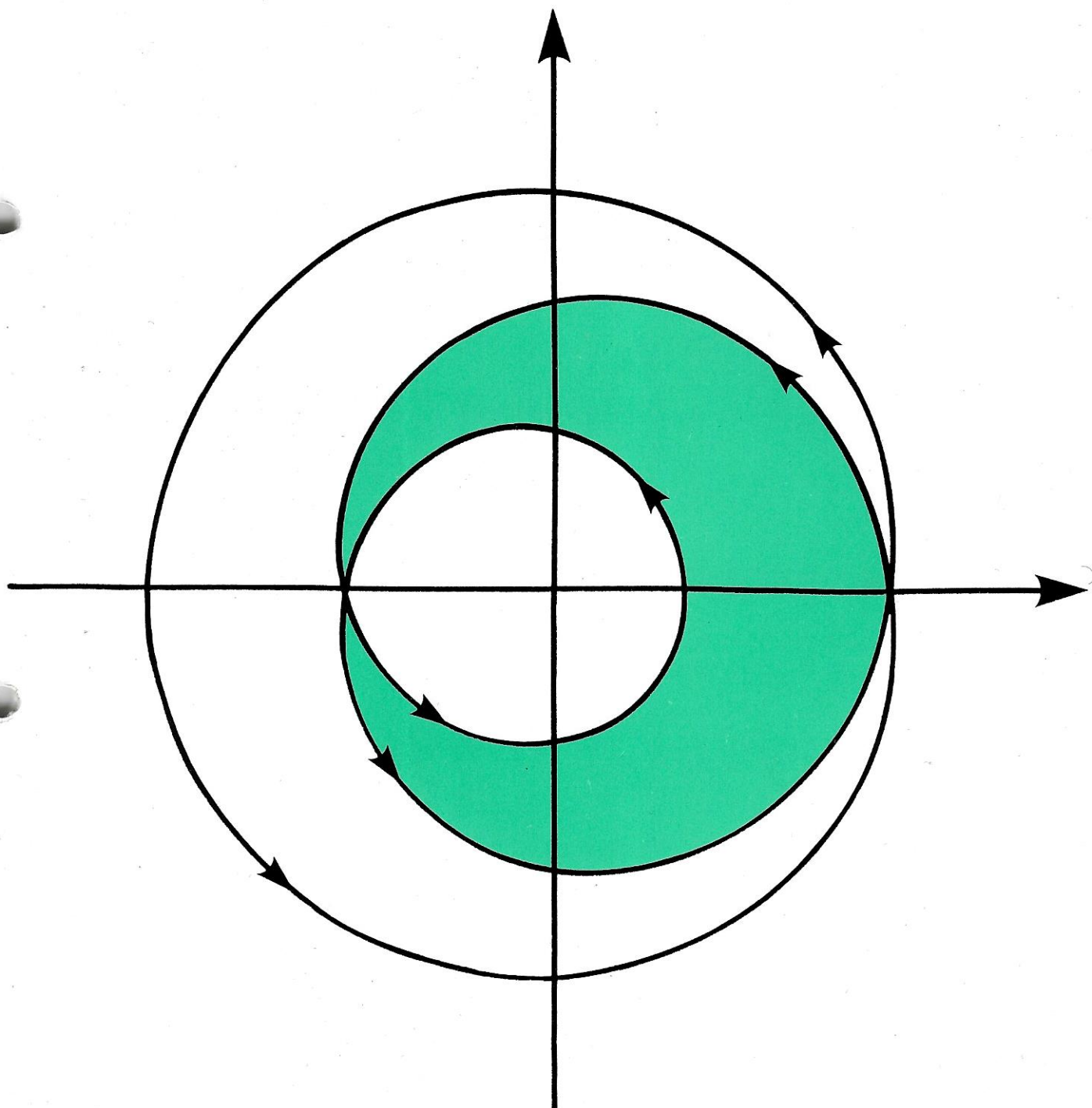
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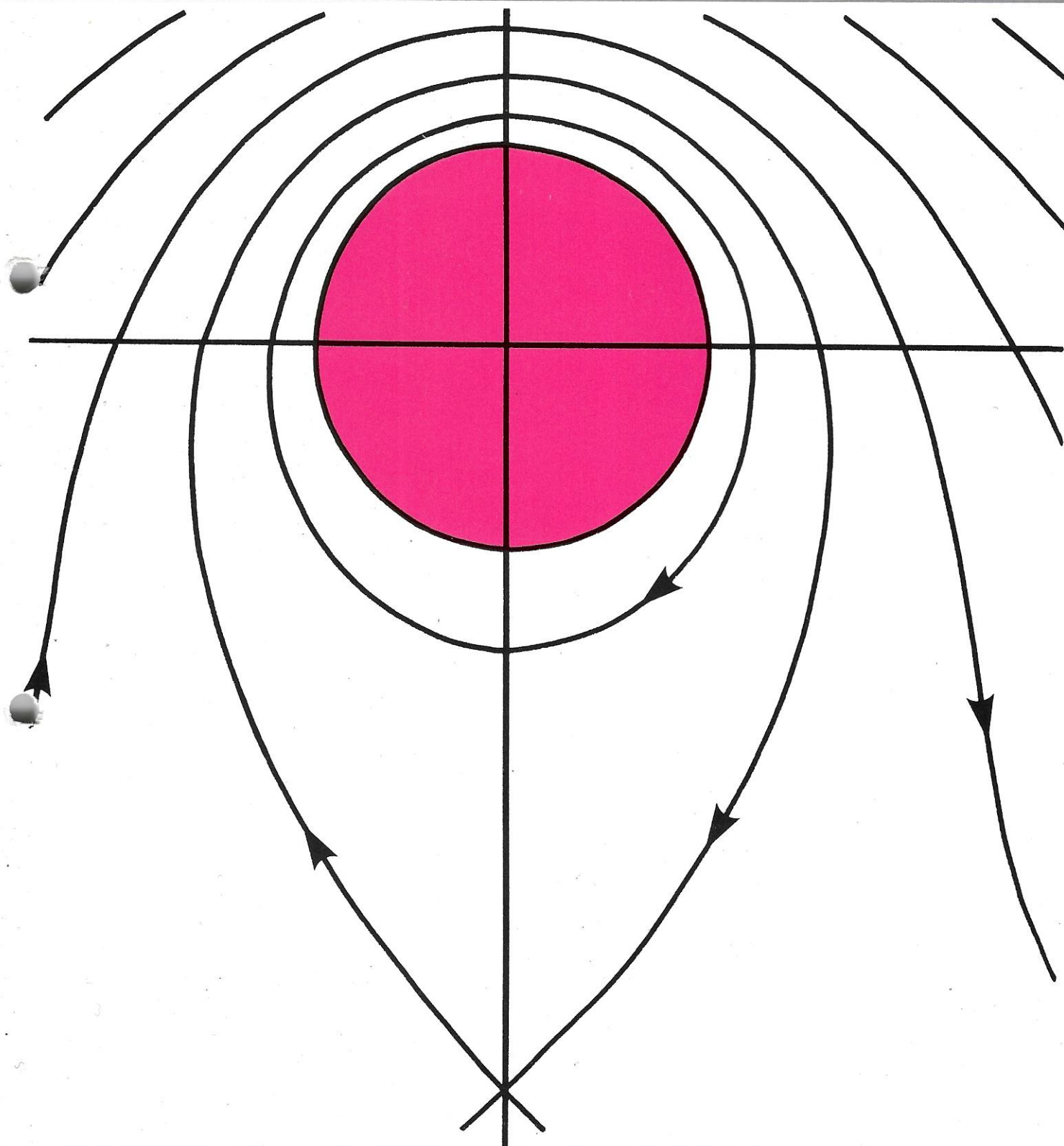
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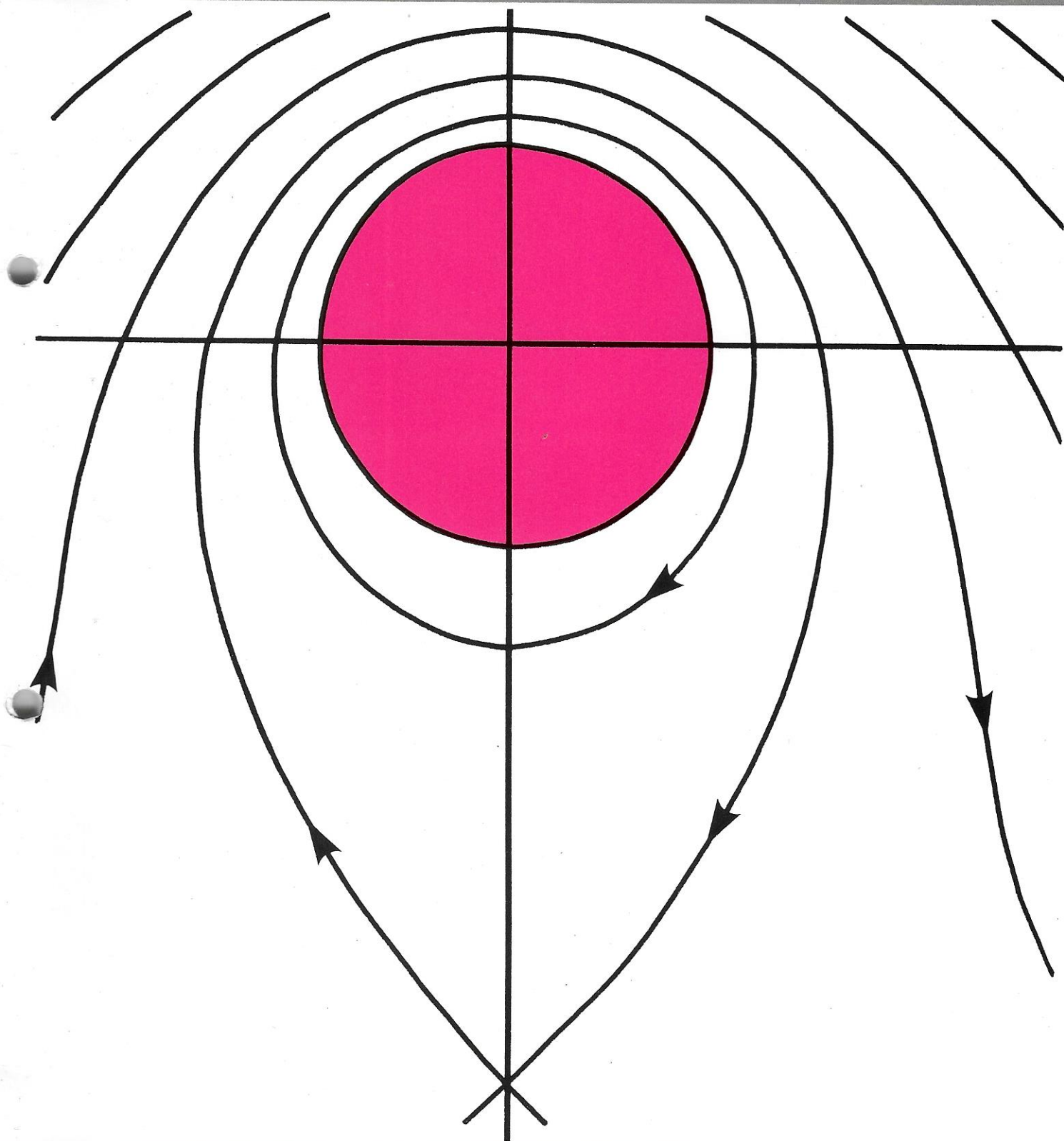
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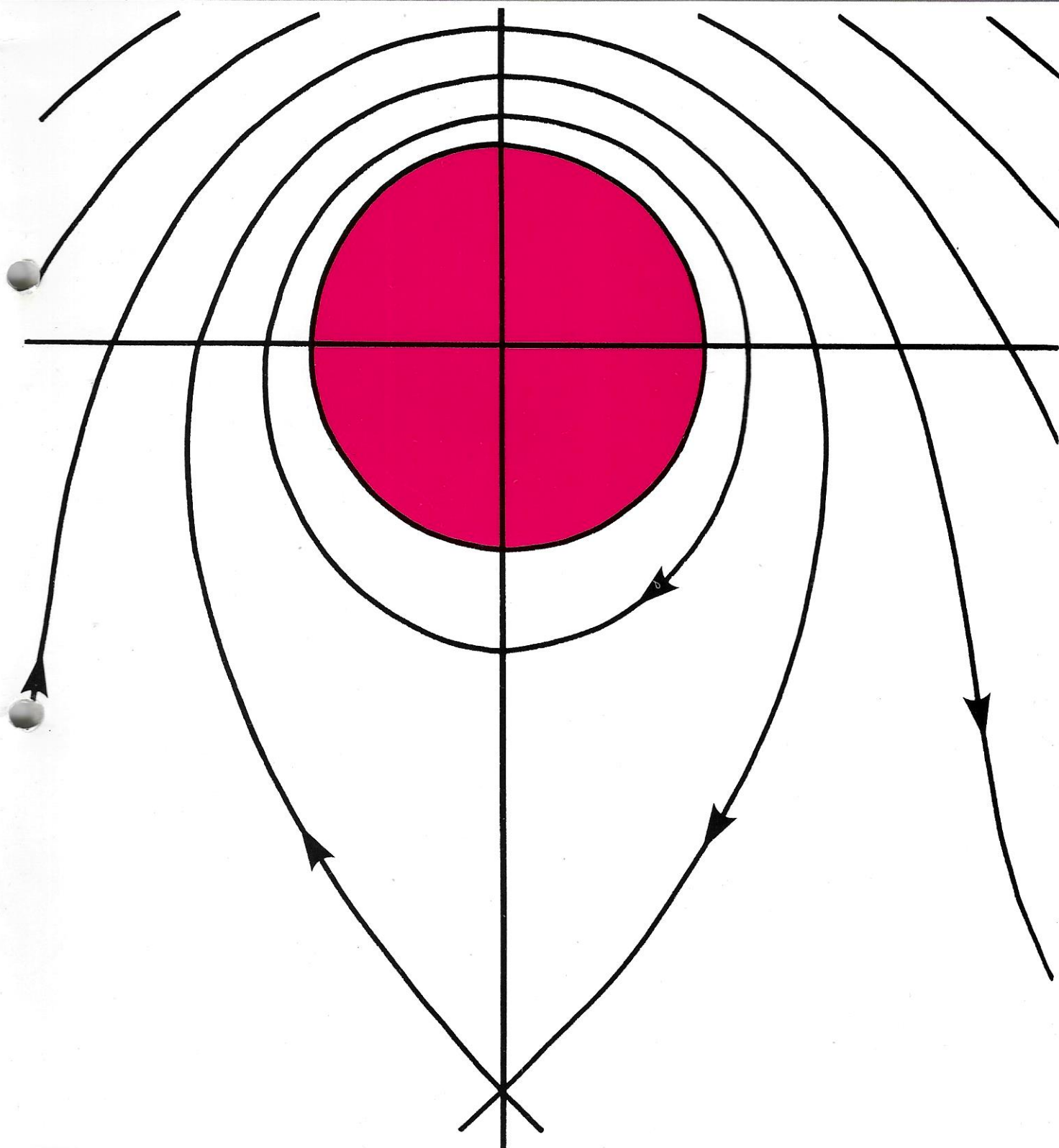
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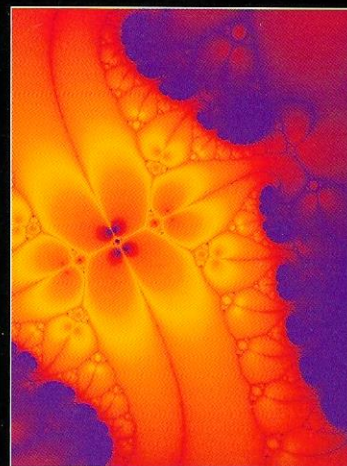




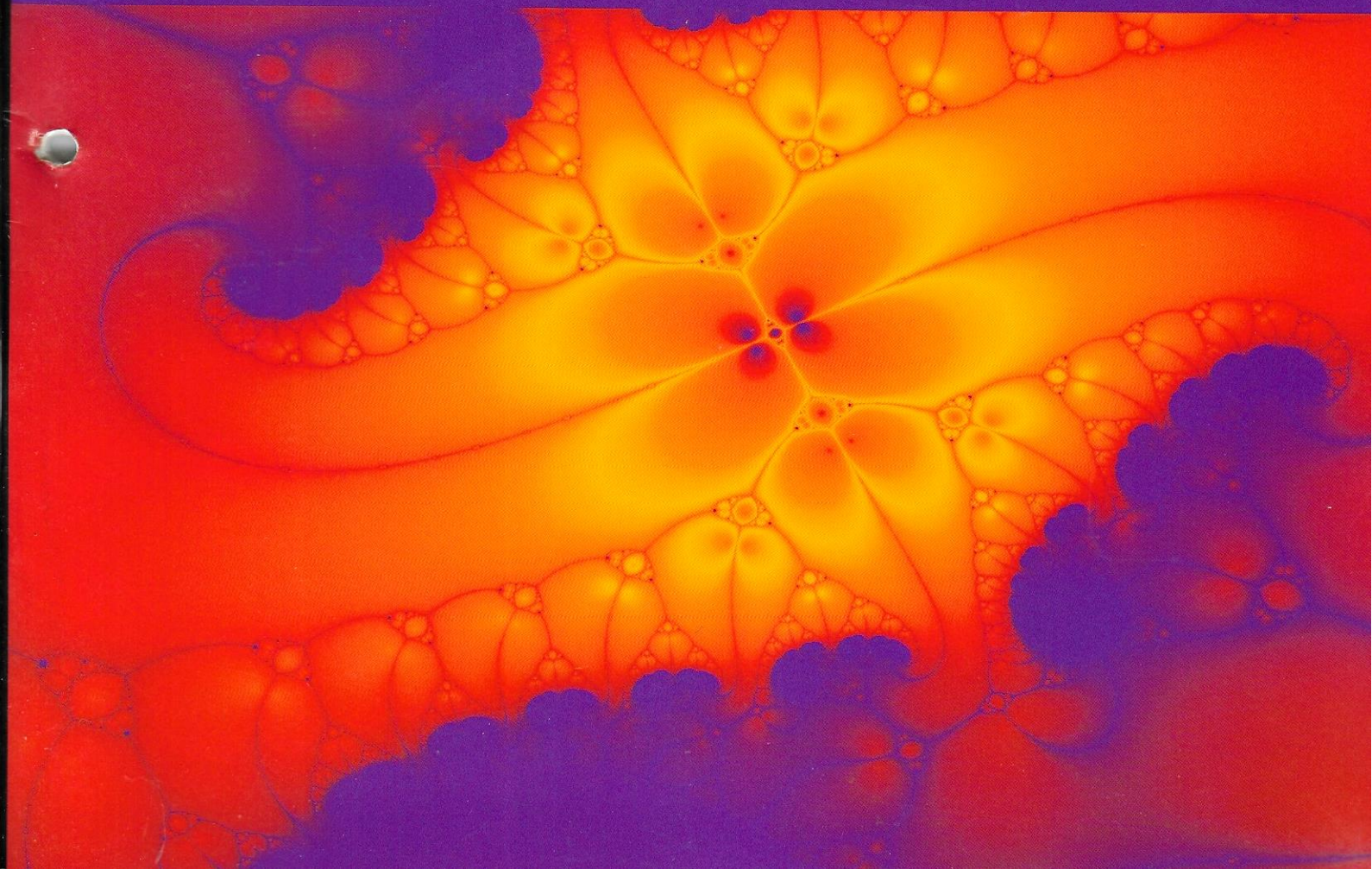
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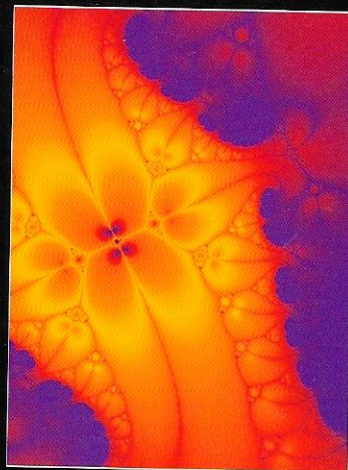




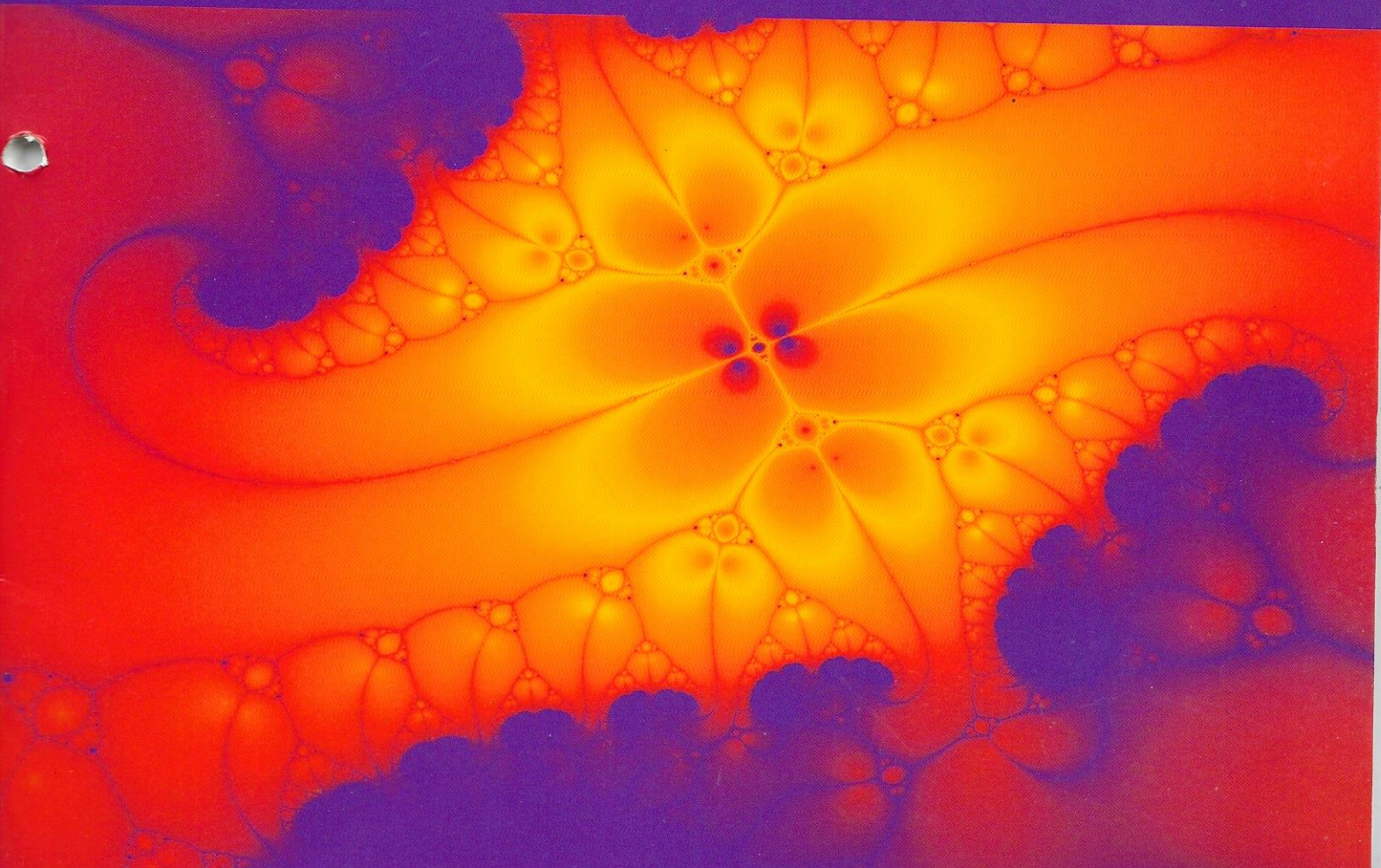
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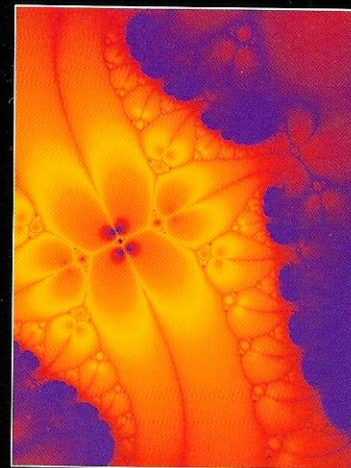




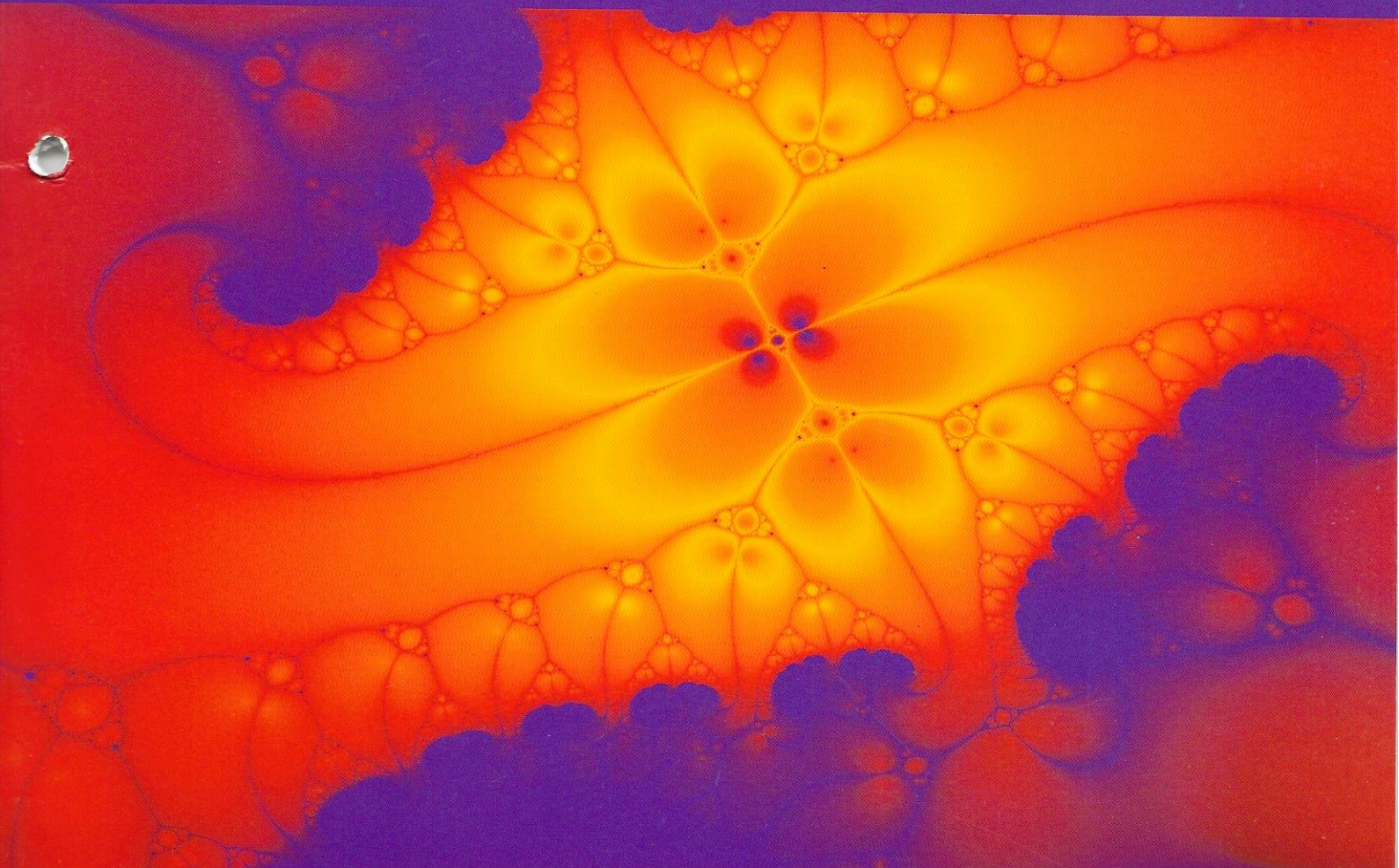
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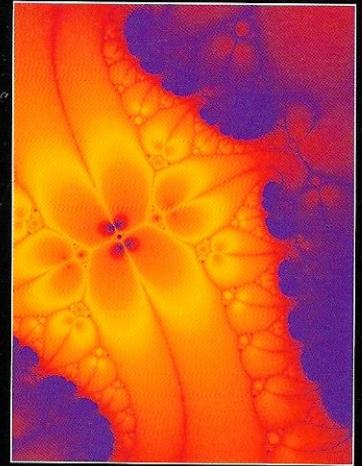




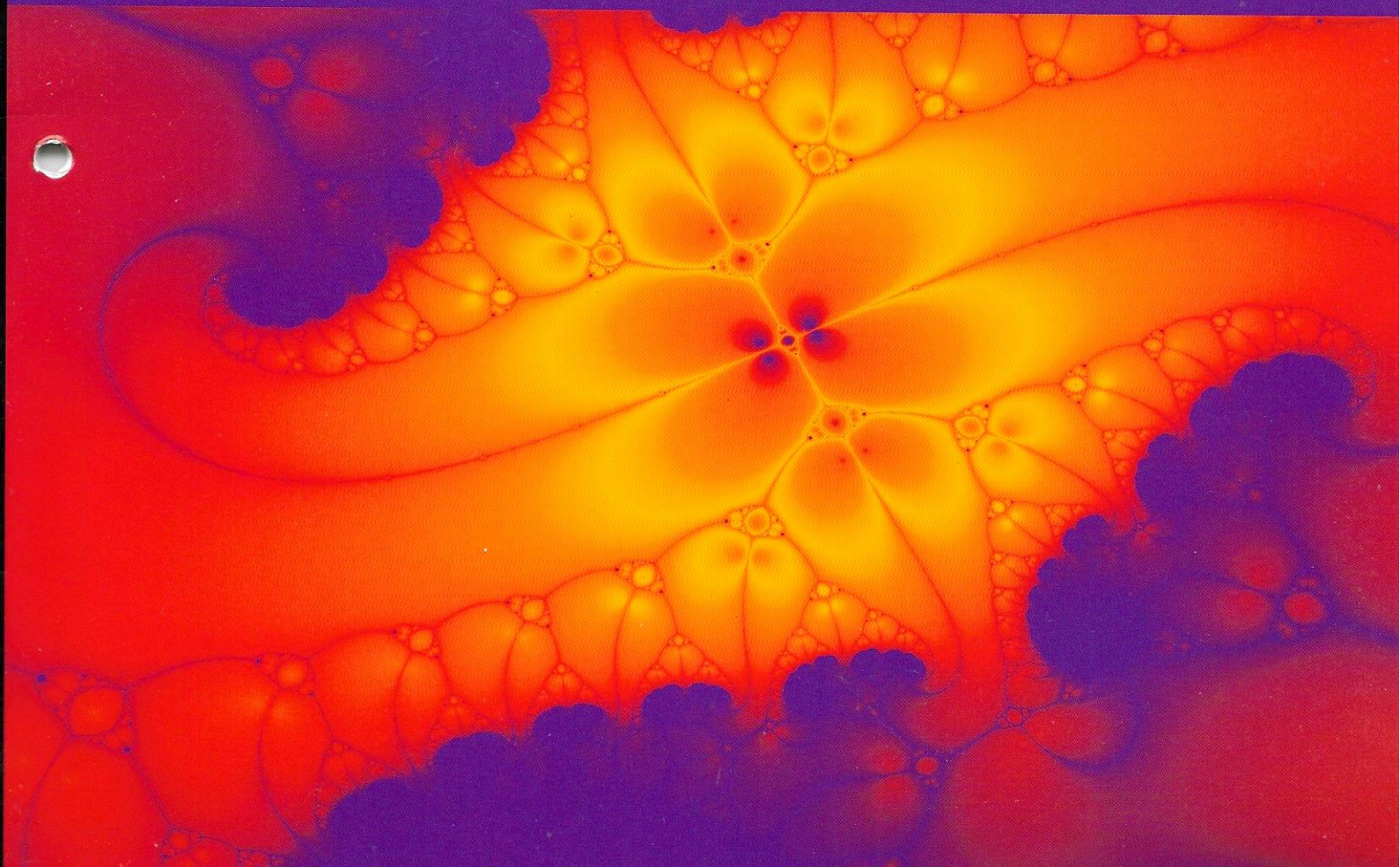
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Unit A4 Closed sets

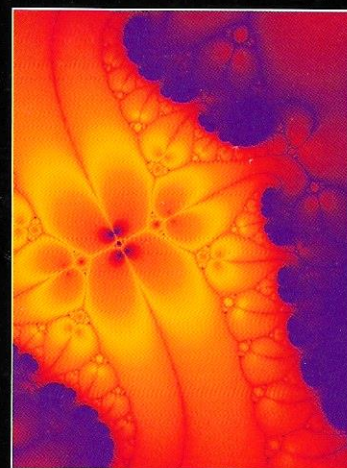




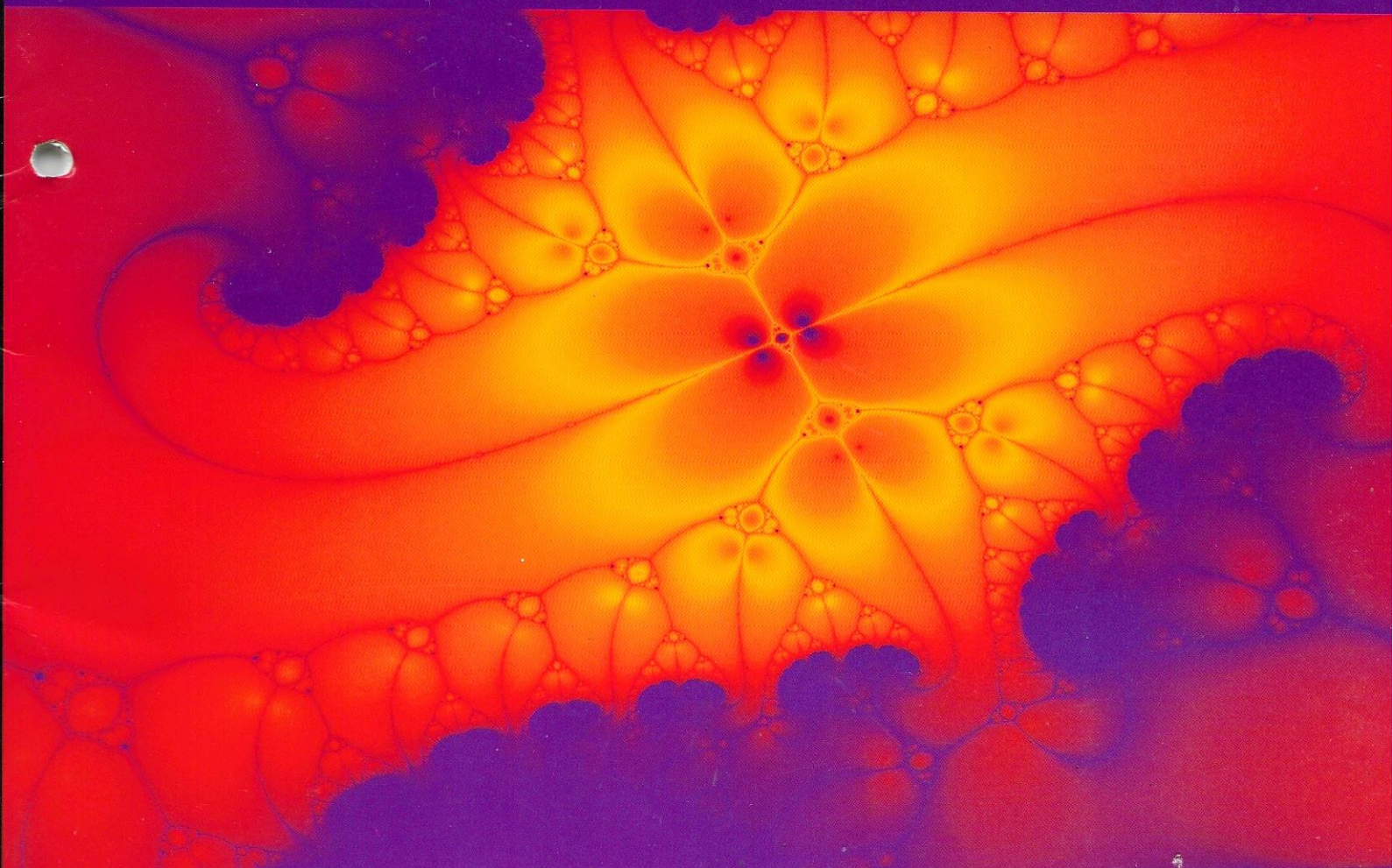
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Unit B1 Surfaces

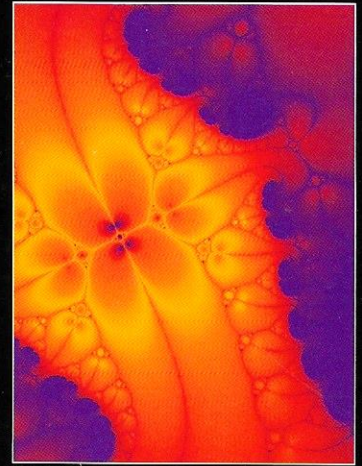




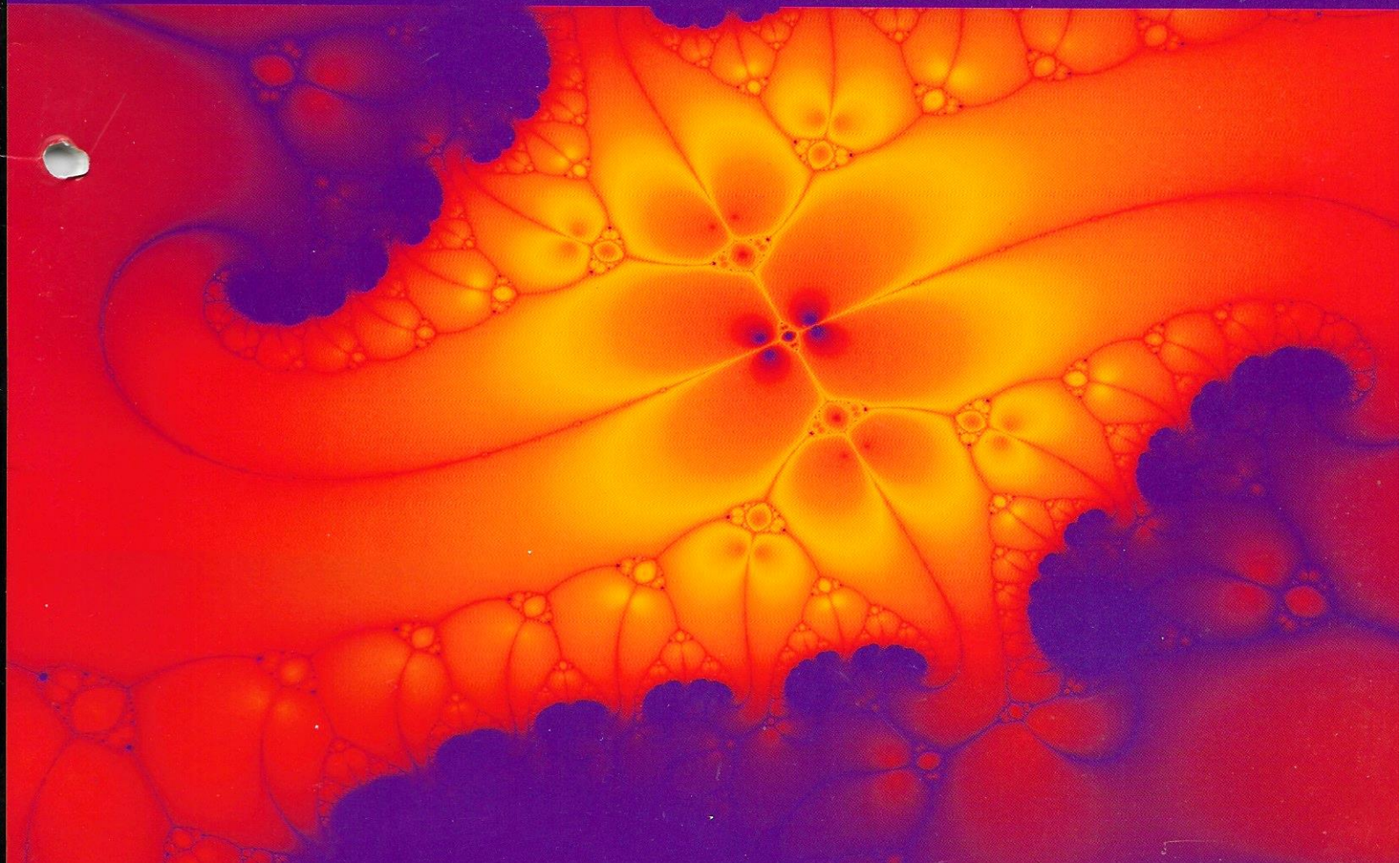
The Open University

M338 Topology

B2



Unit B2 Subdivisions

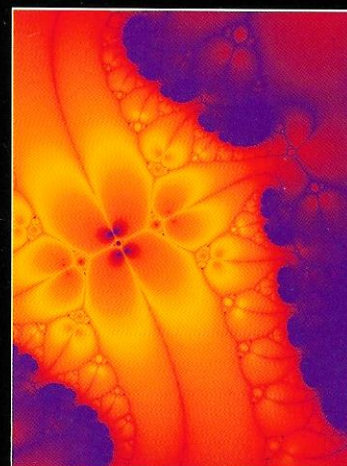




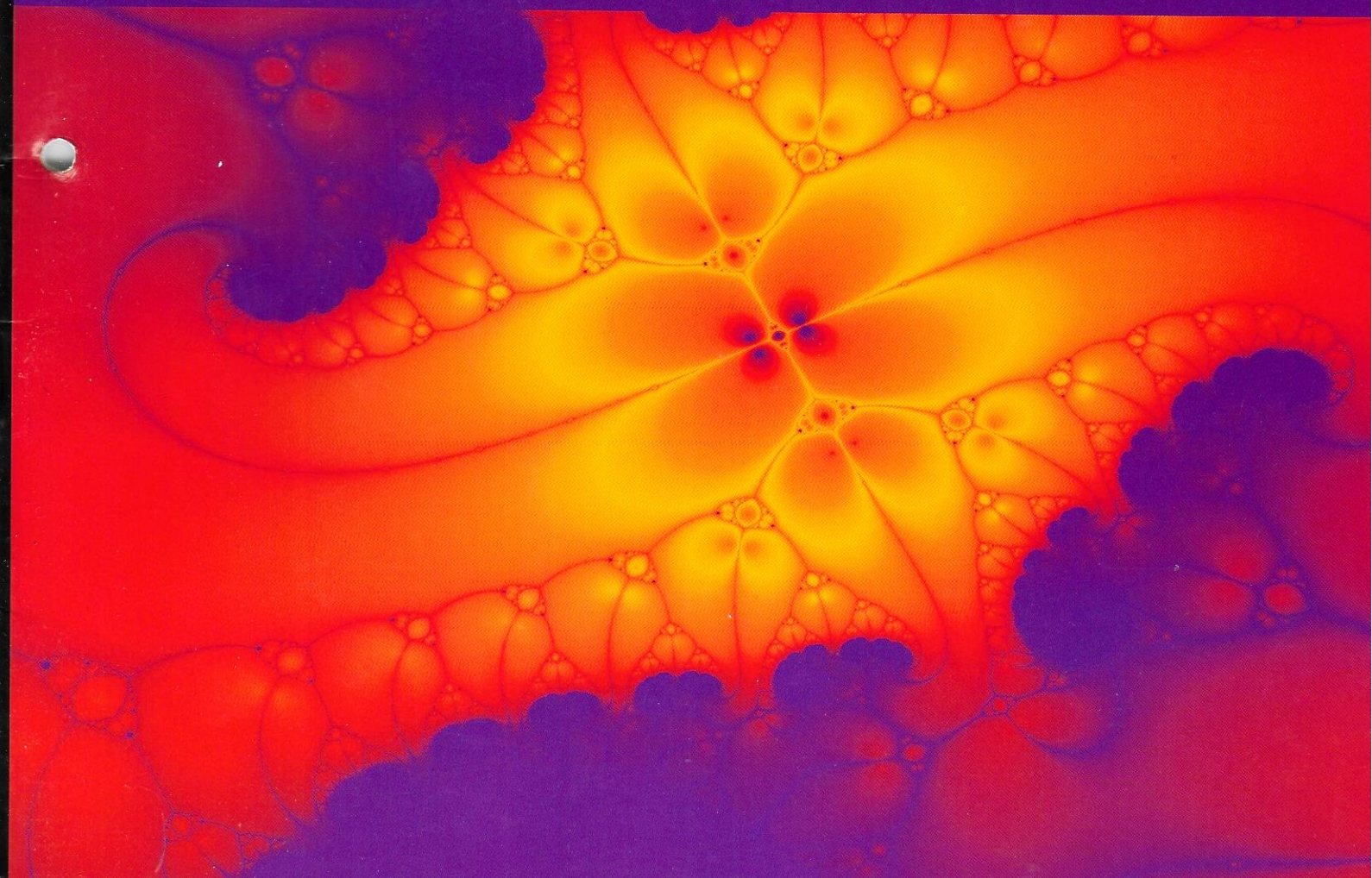
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M338 Topology

B3



Unit B3 Classification of surfaces

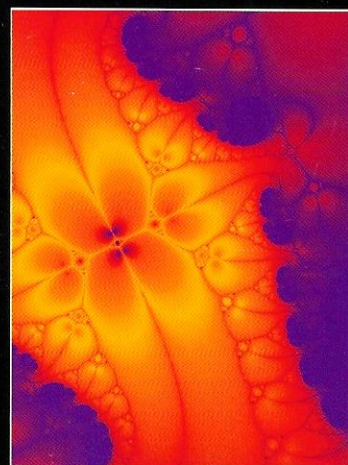




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B4



Unit B4 Graphs on surfaces





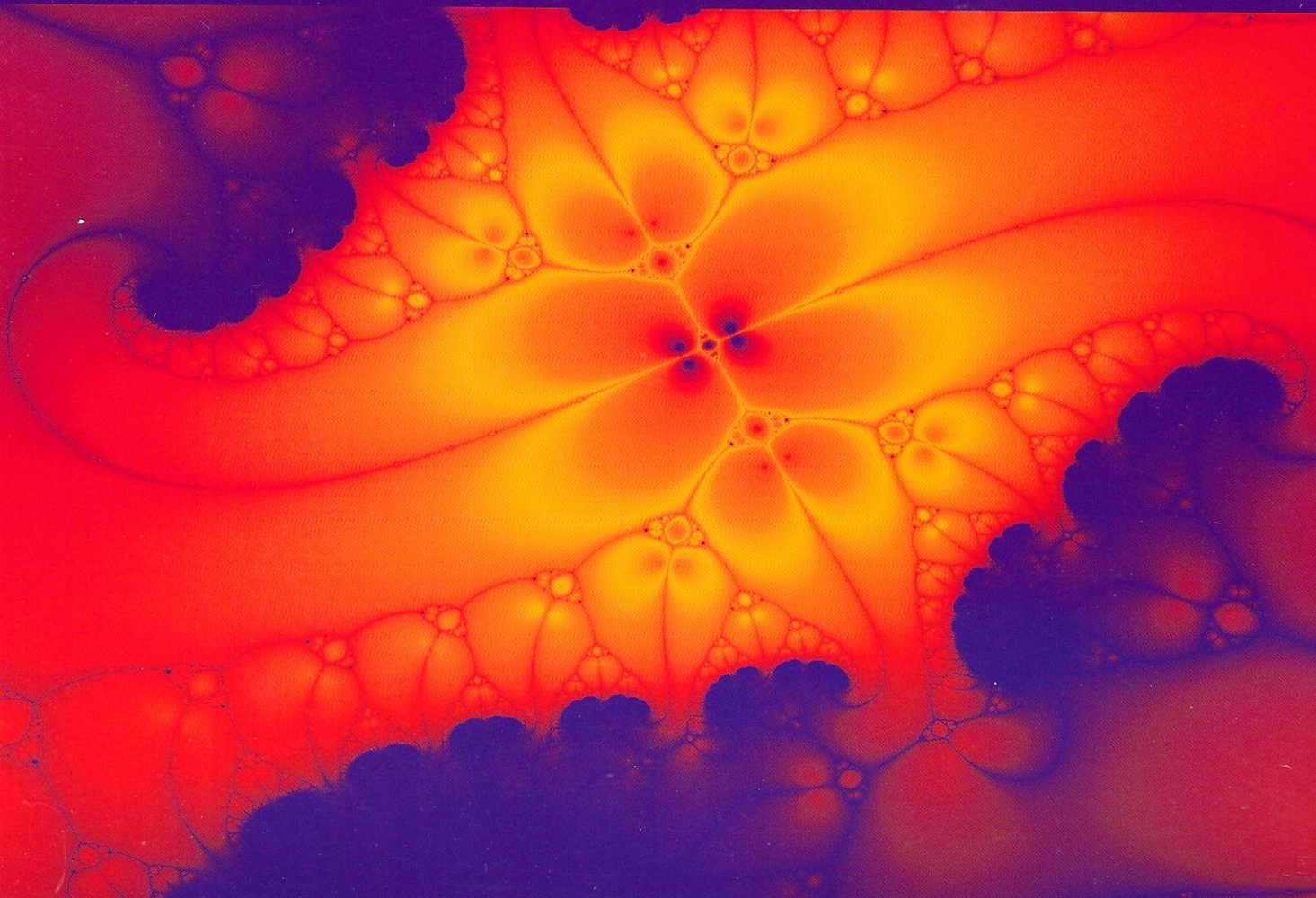
The Open University

M338 Topology

C1



Unit C1 Connectedness





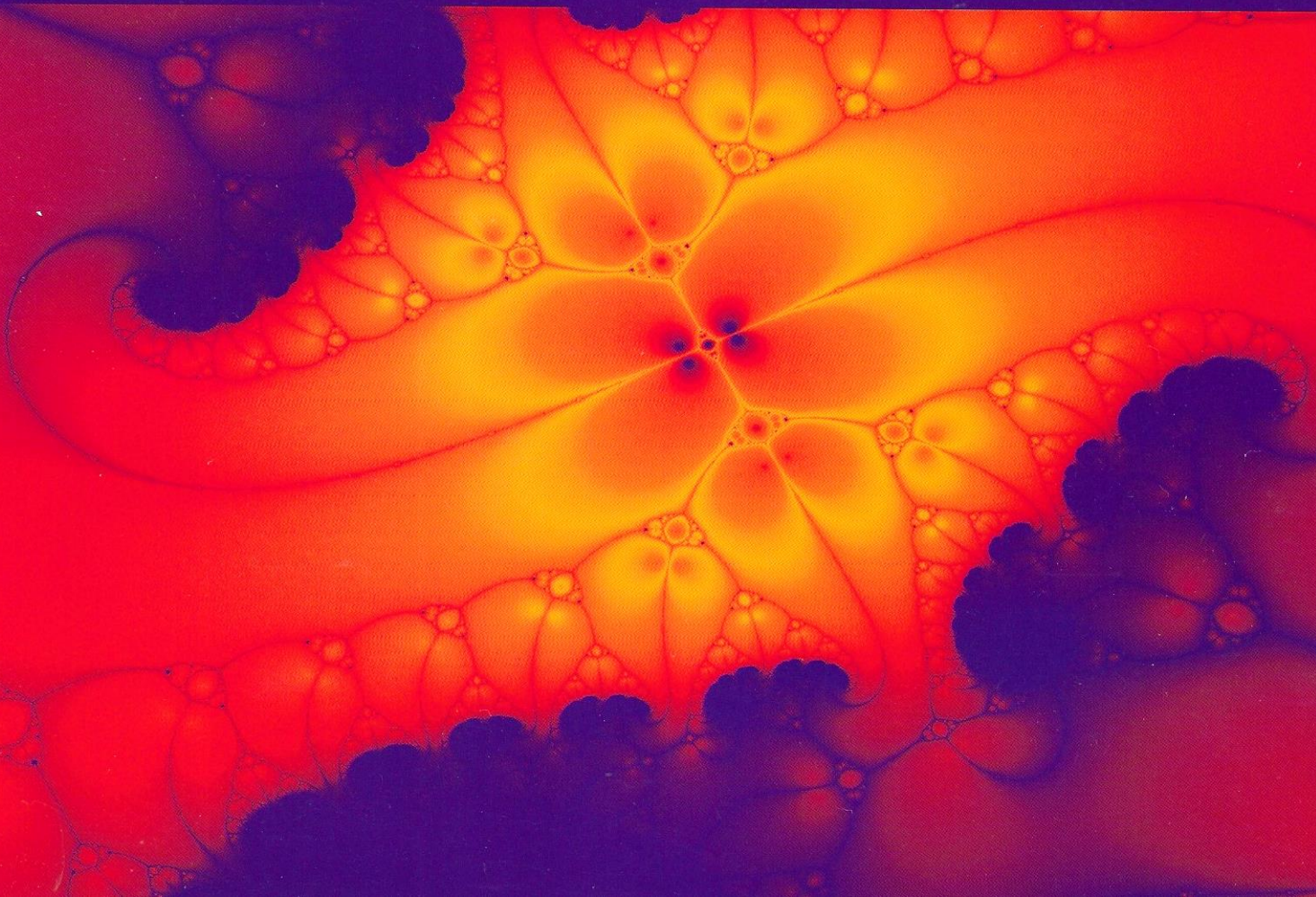
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M338 Topology

C2



Unit C2 Compactness

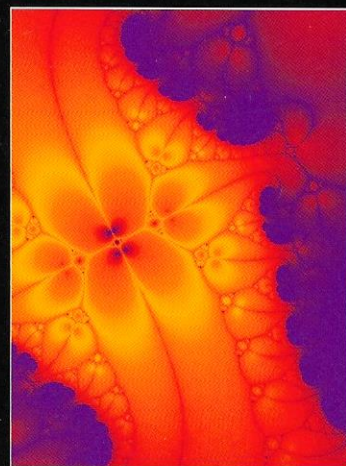




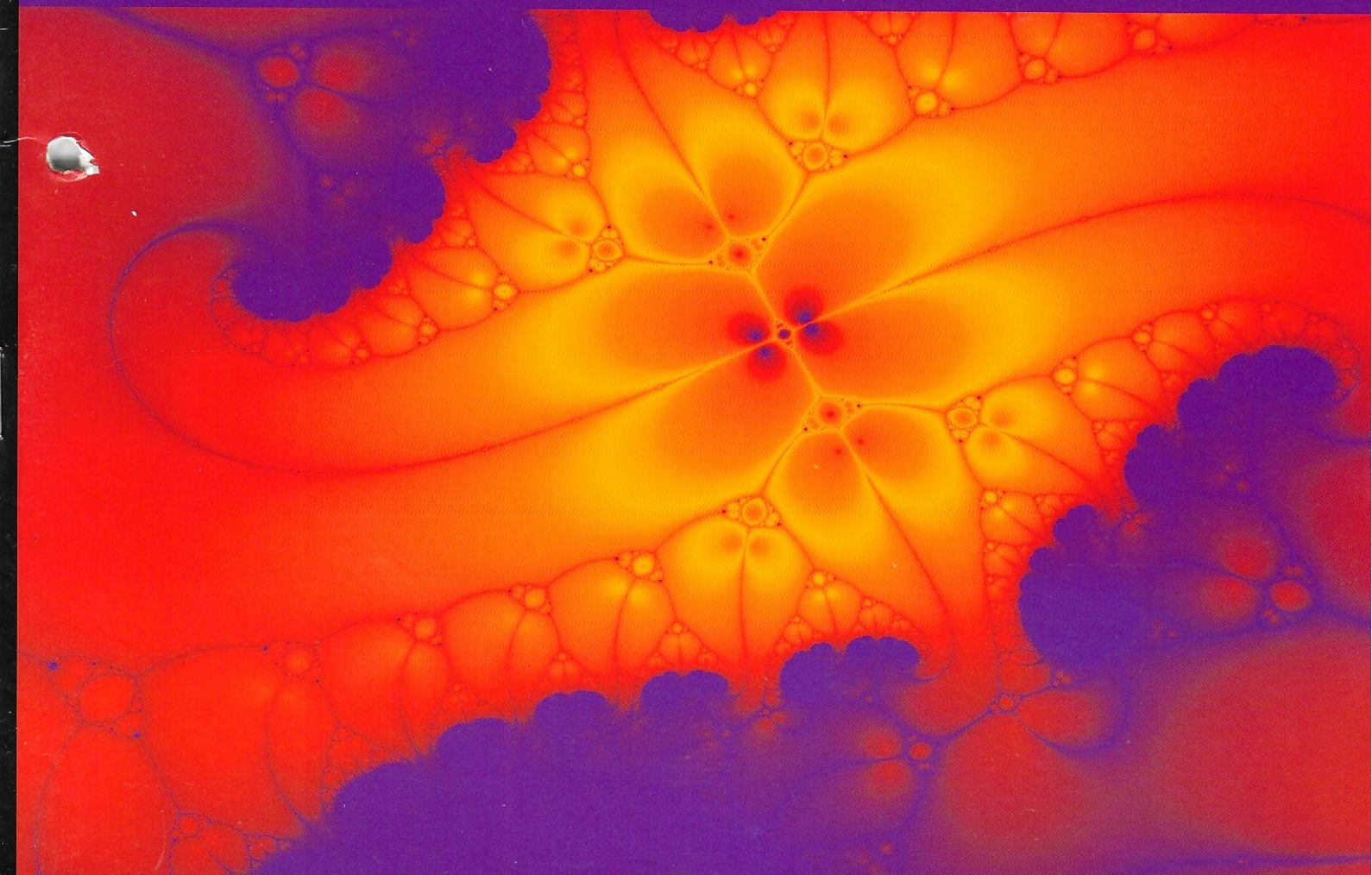
The Open University

M338 Topology

C3



Unit C3 Sequences

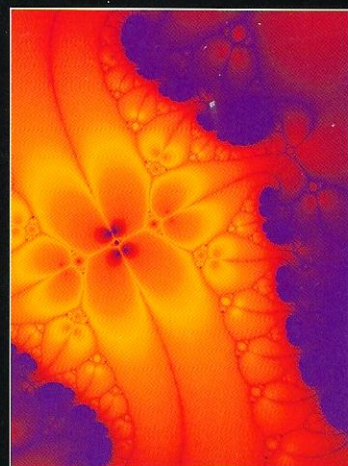




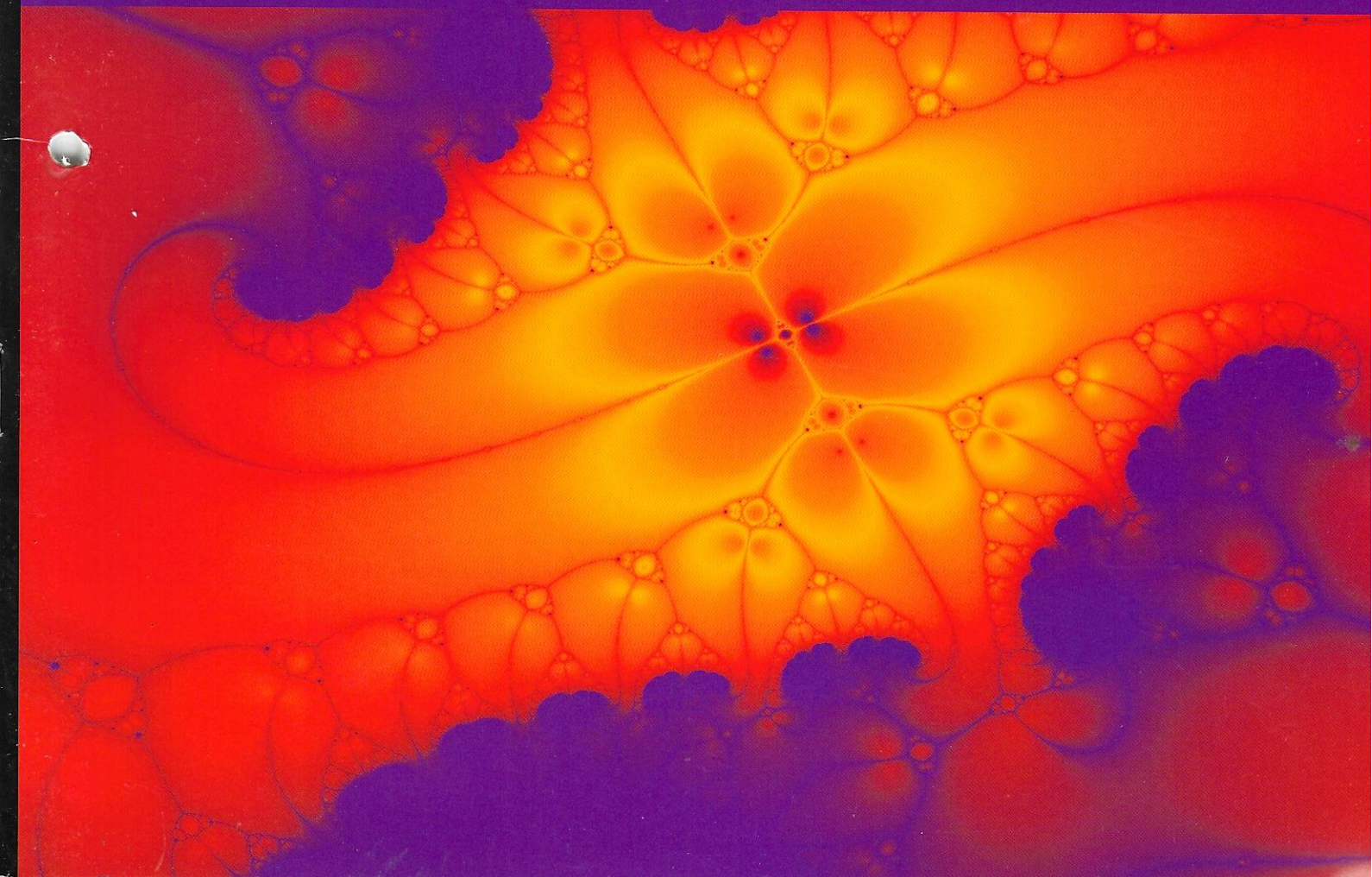
The Open University

M338 Topology

C4



Unit C4 Completeness





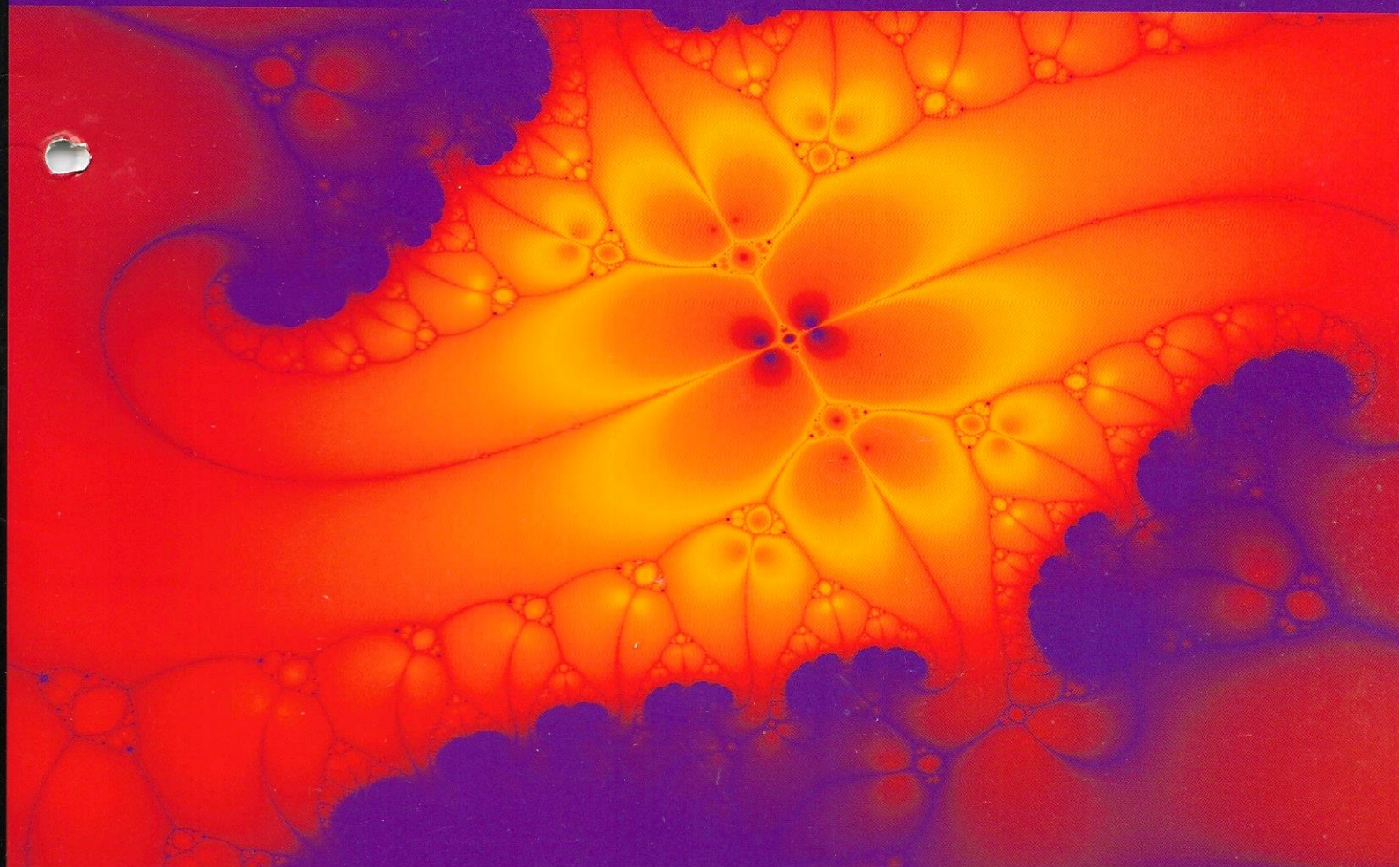
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M338 Topology

C5



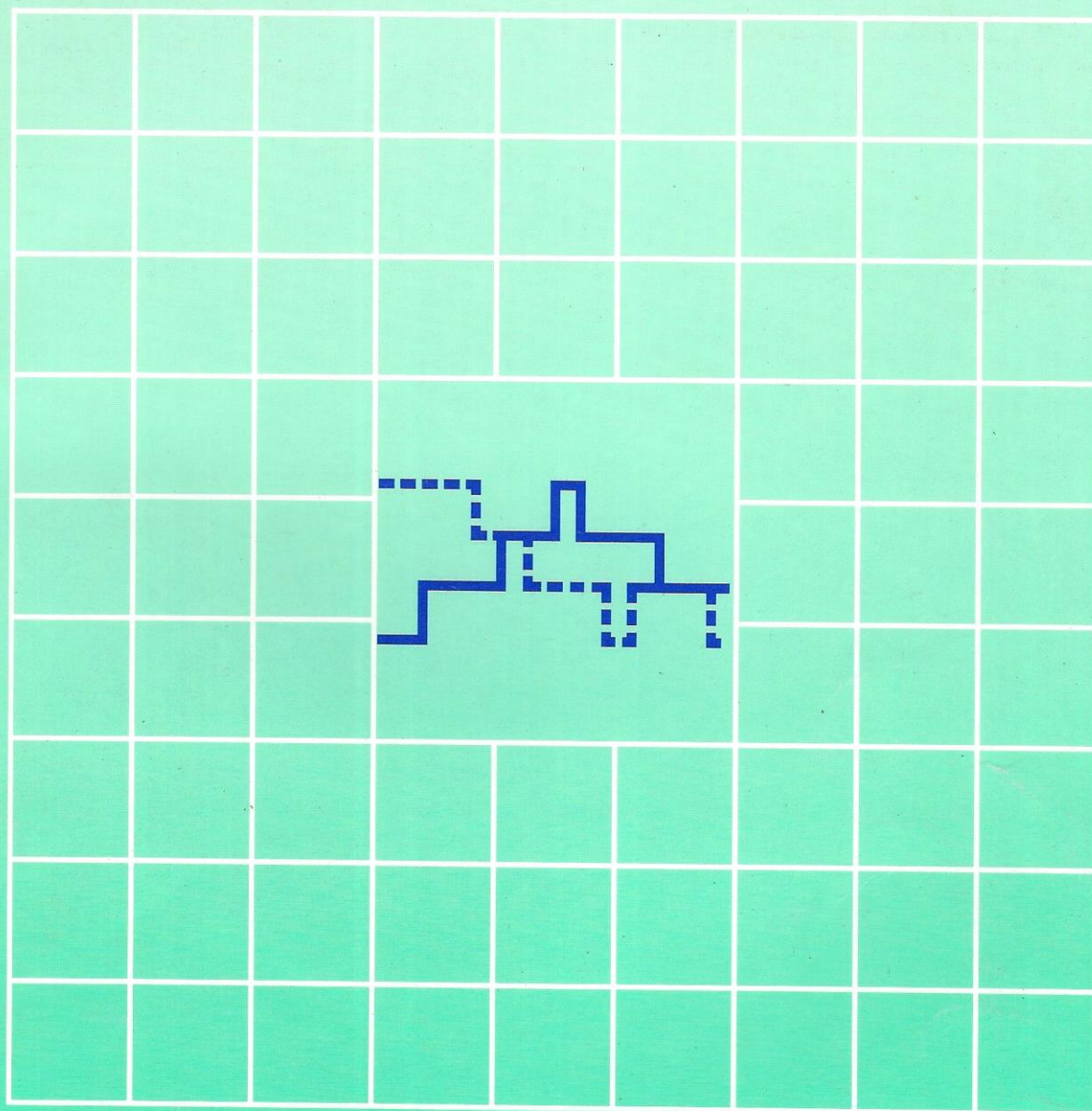
Unit C5 Fractals





Unit 2

Random processes

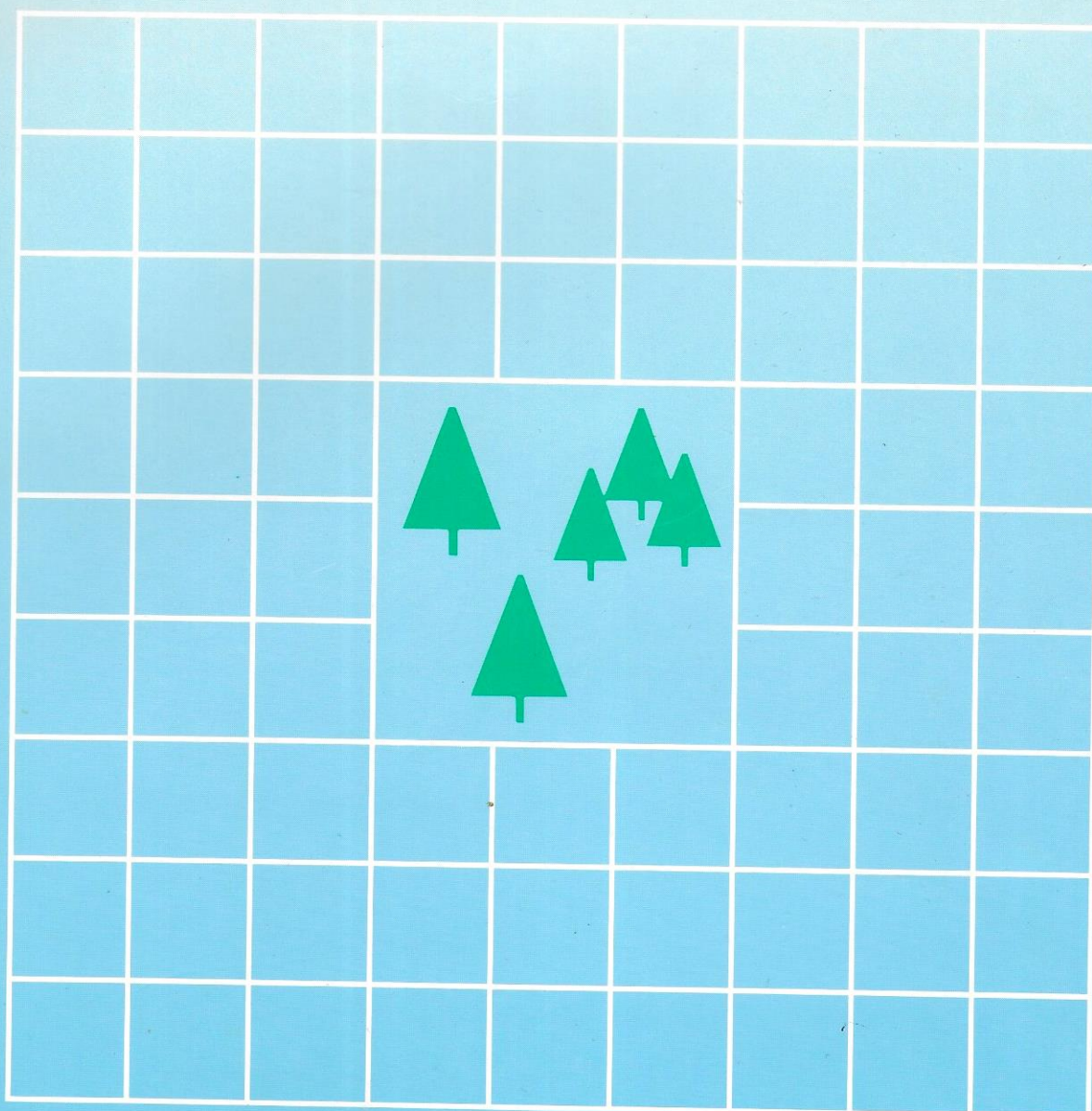


M343 APPLICATIONS OF PROBABILITY



Unit 3

Patterns in space

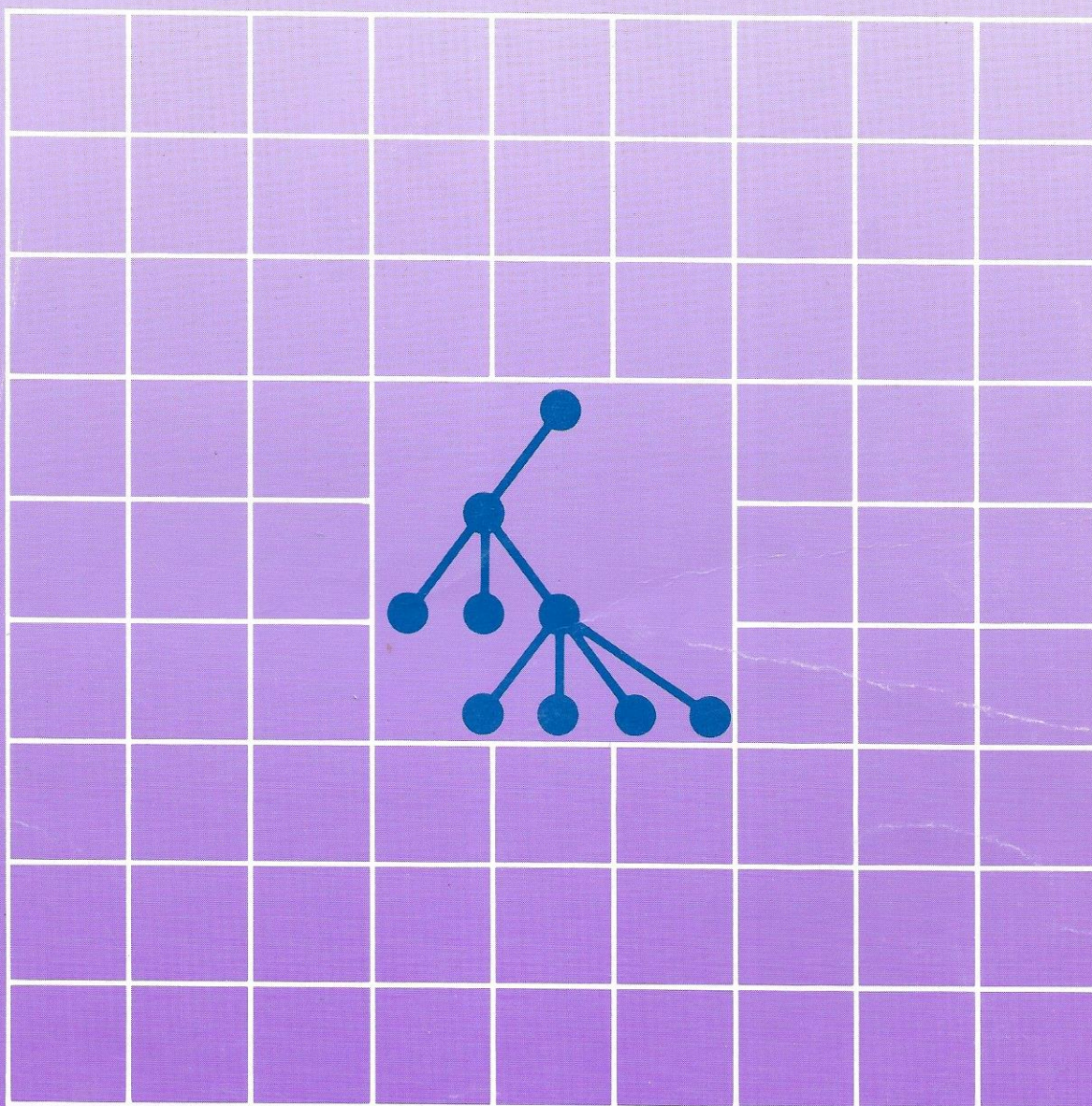


M343 APPLICATIONS OF PROBABILITY



Unit 4

Branching processes

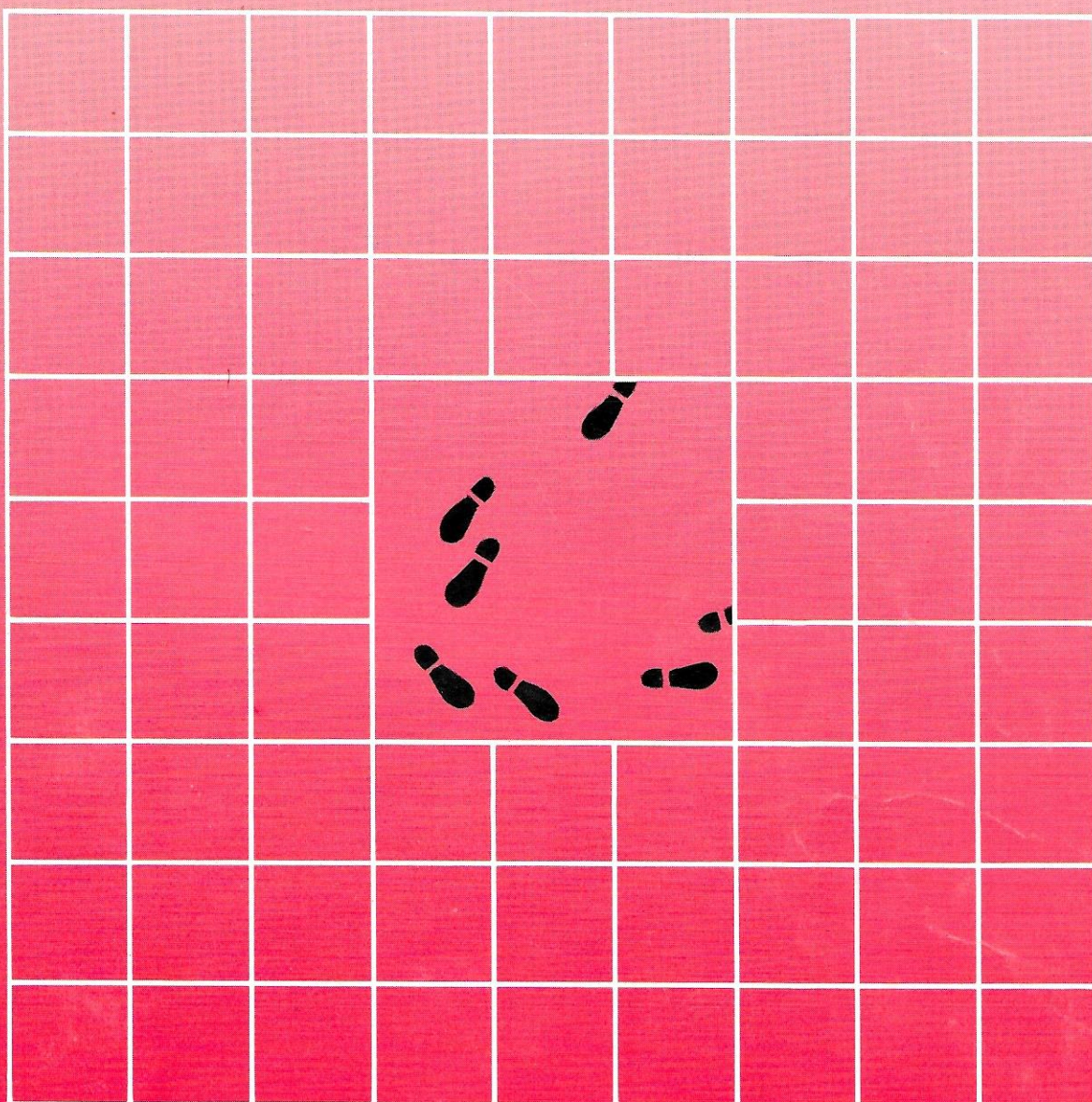


M343 APPLICATIONS OF PROBABILITY



Unit 5

Random walks



M343 APPLICATIONS OF PROBABILITY



Unit 6

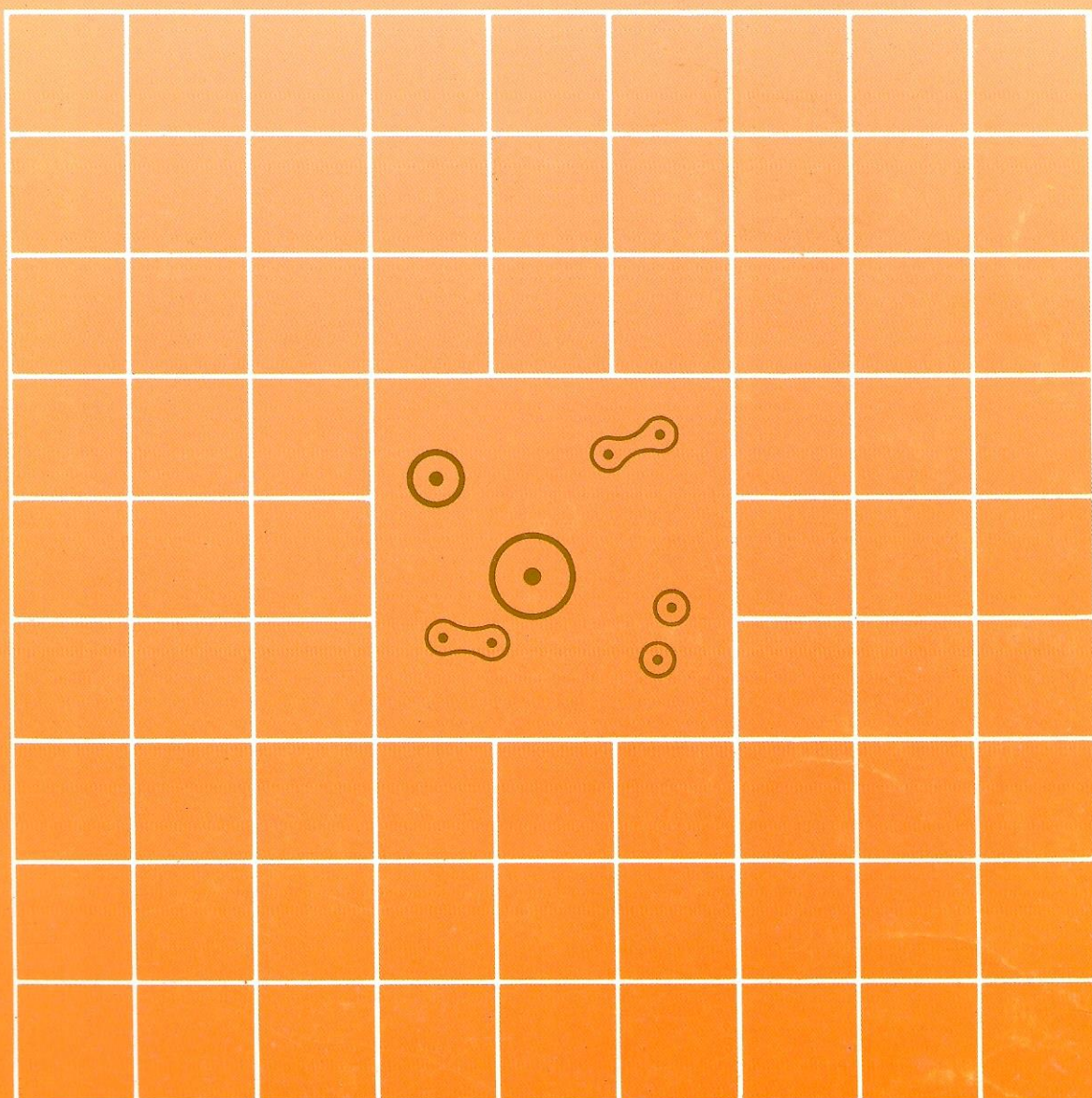
Markov chains





Unit 7

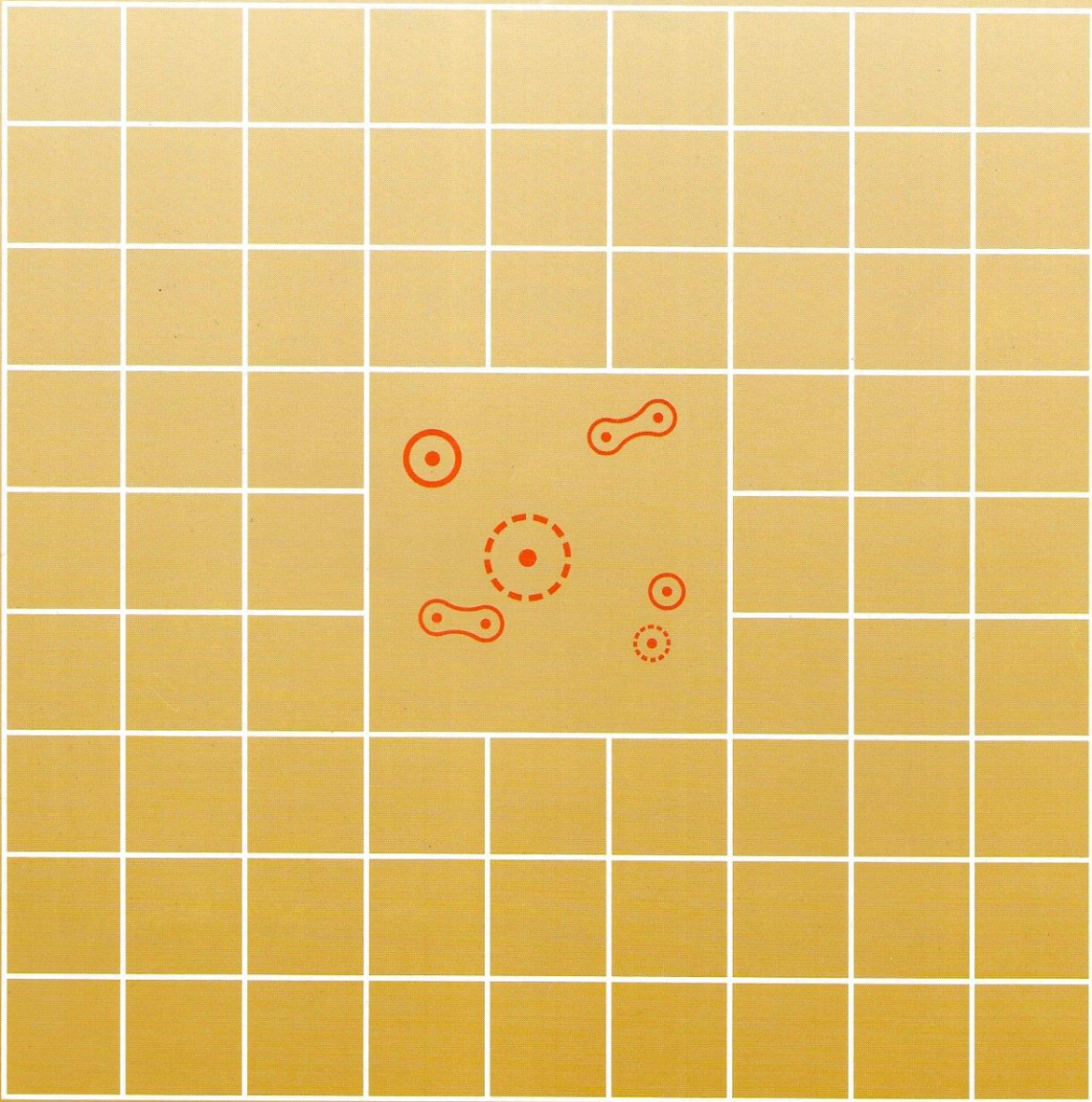
Birth processes





Unit 8

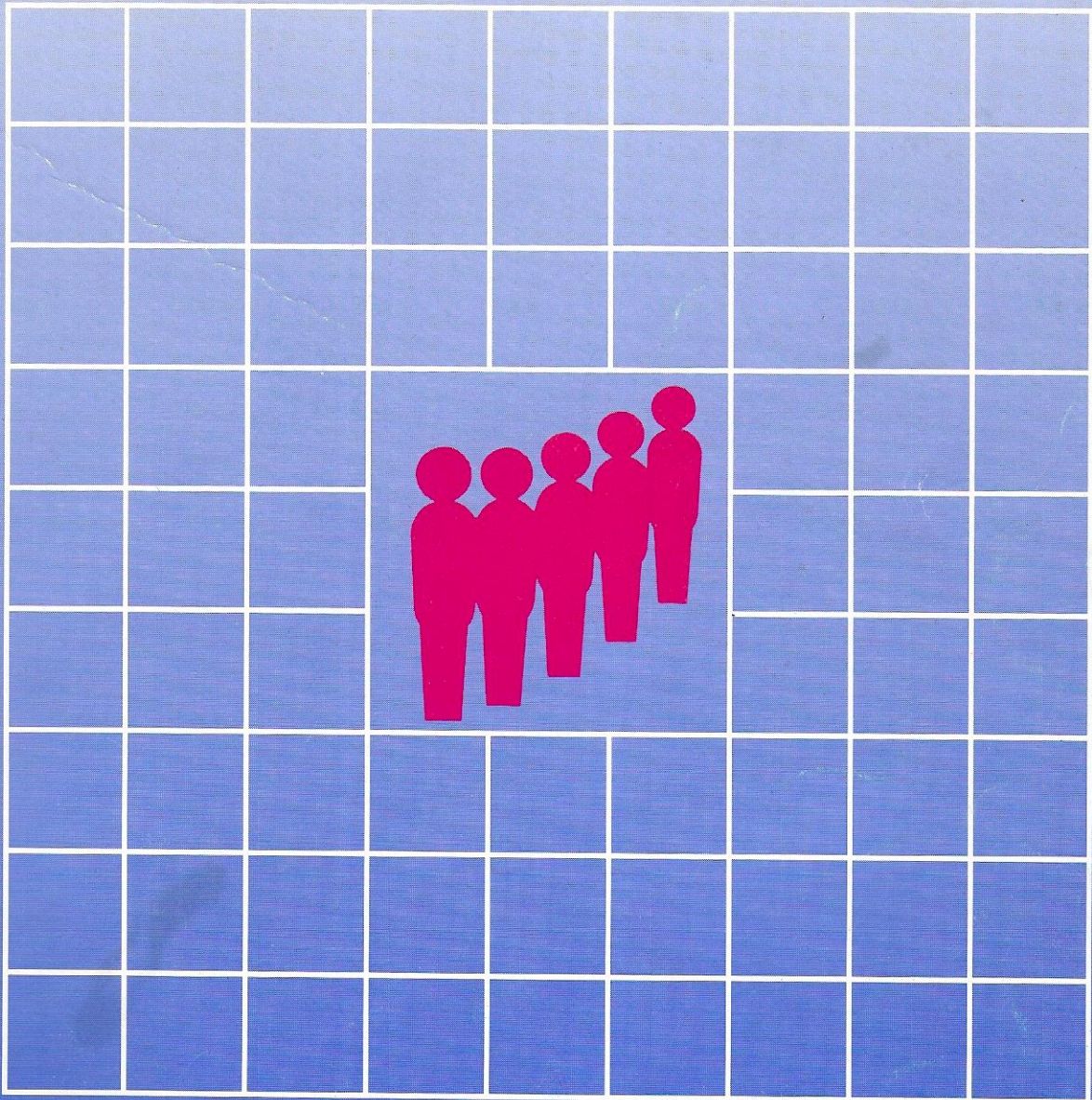
Birth and death processes





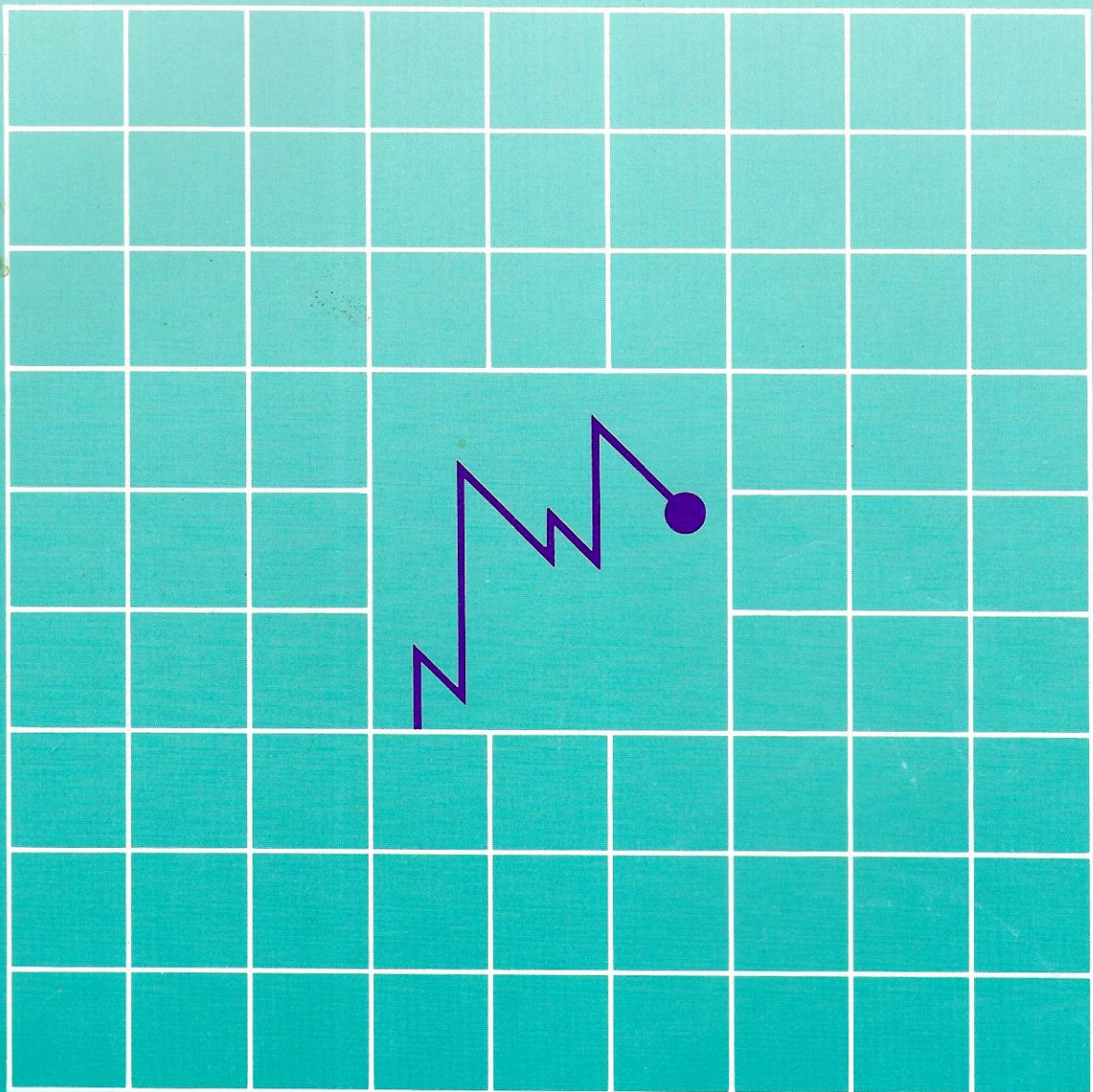
Unit 9

Queues





Unit 10 Epidemics

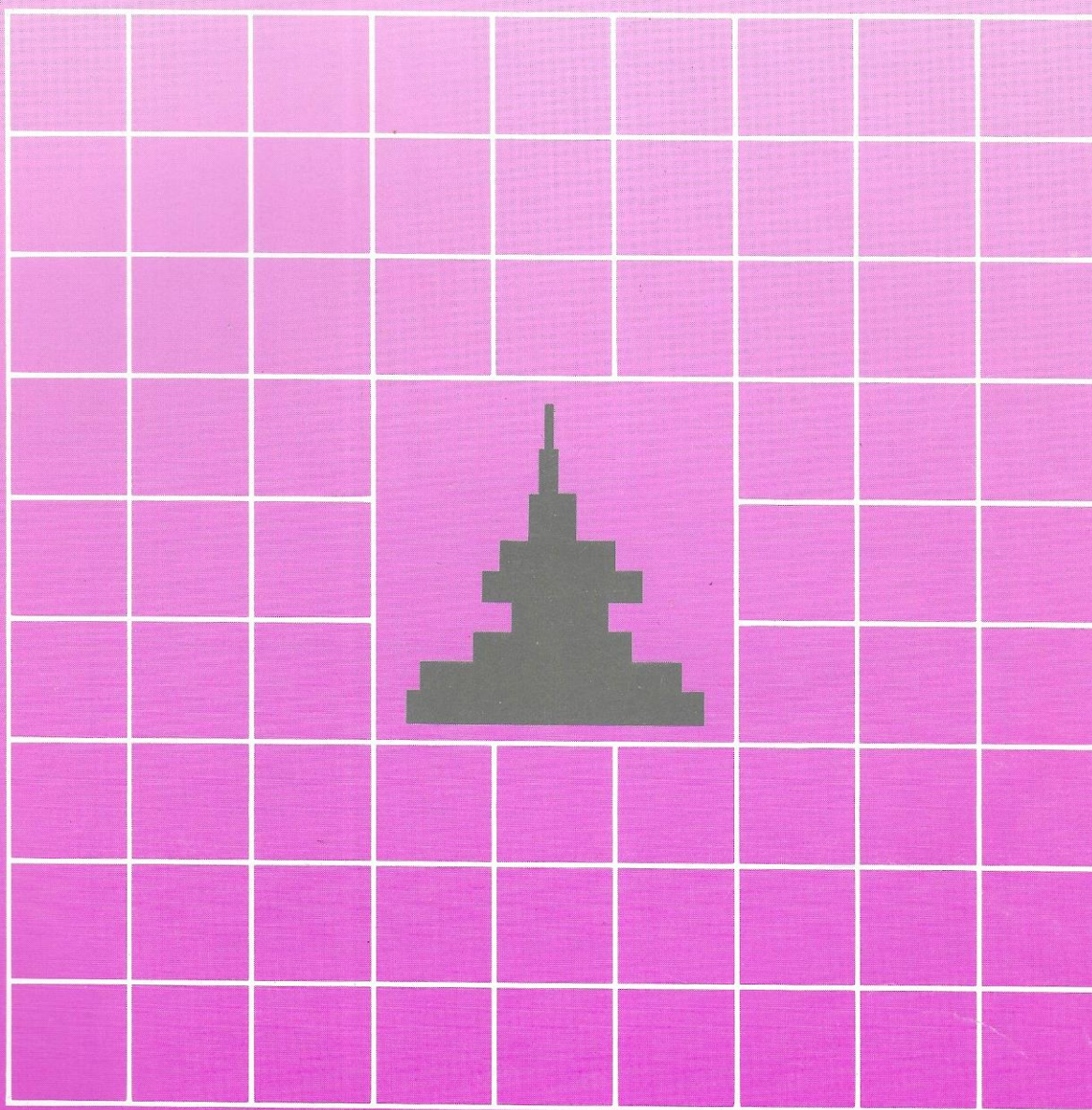


M343 APPLICATIONS OF PROBABILITY



Unit 11

More population models

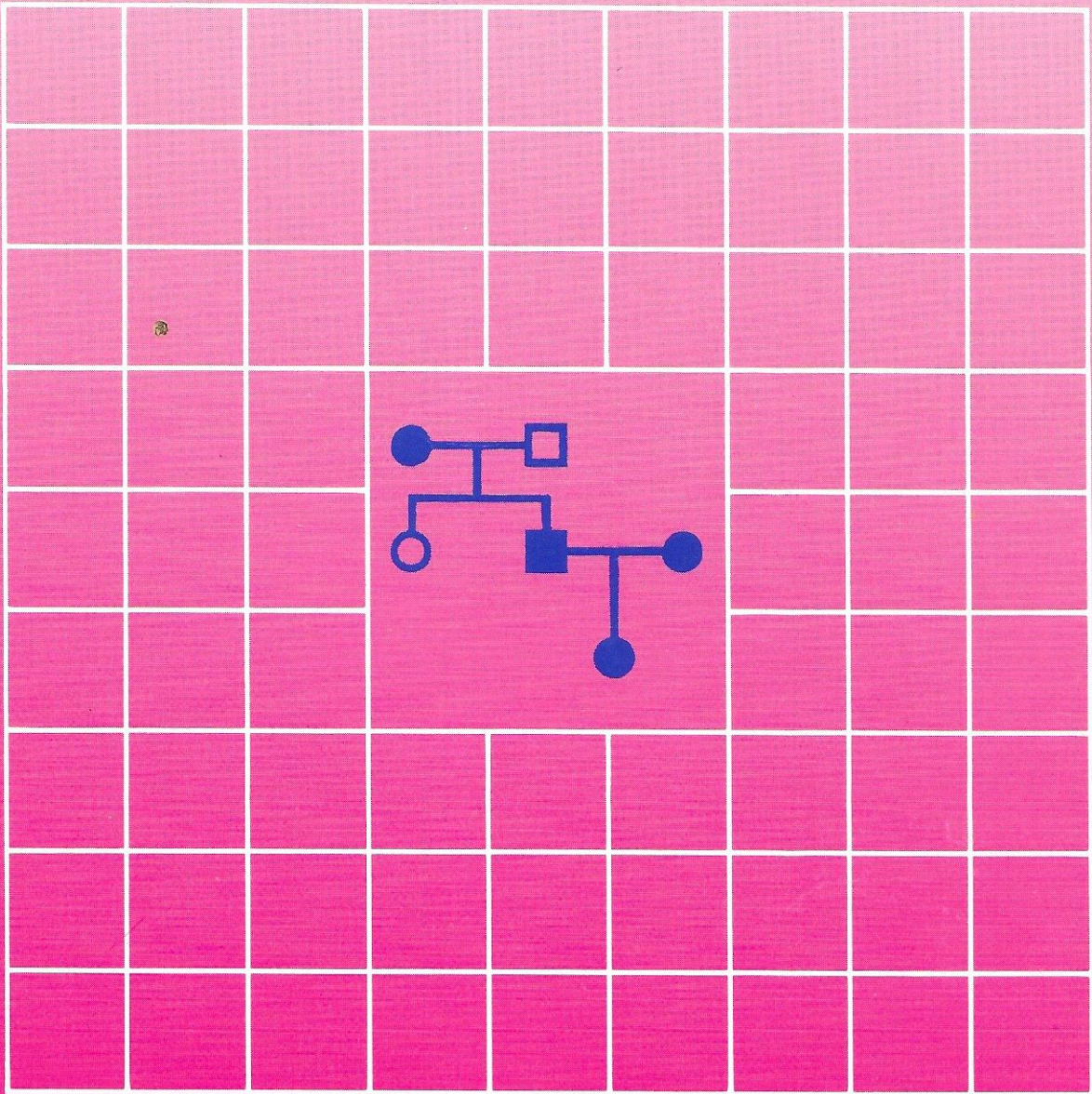


M343 APPLICATIONS OF PROBABILITY



Unit 12

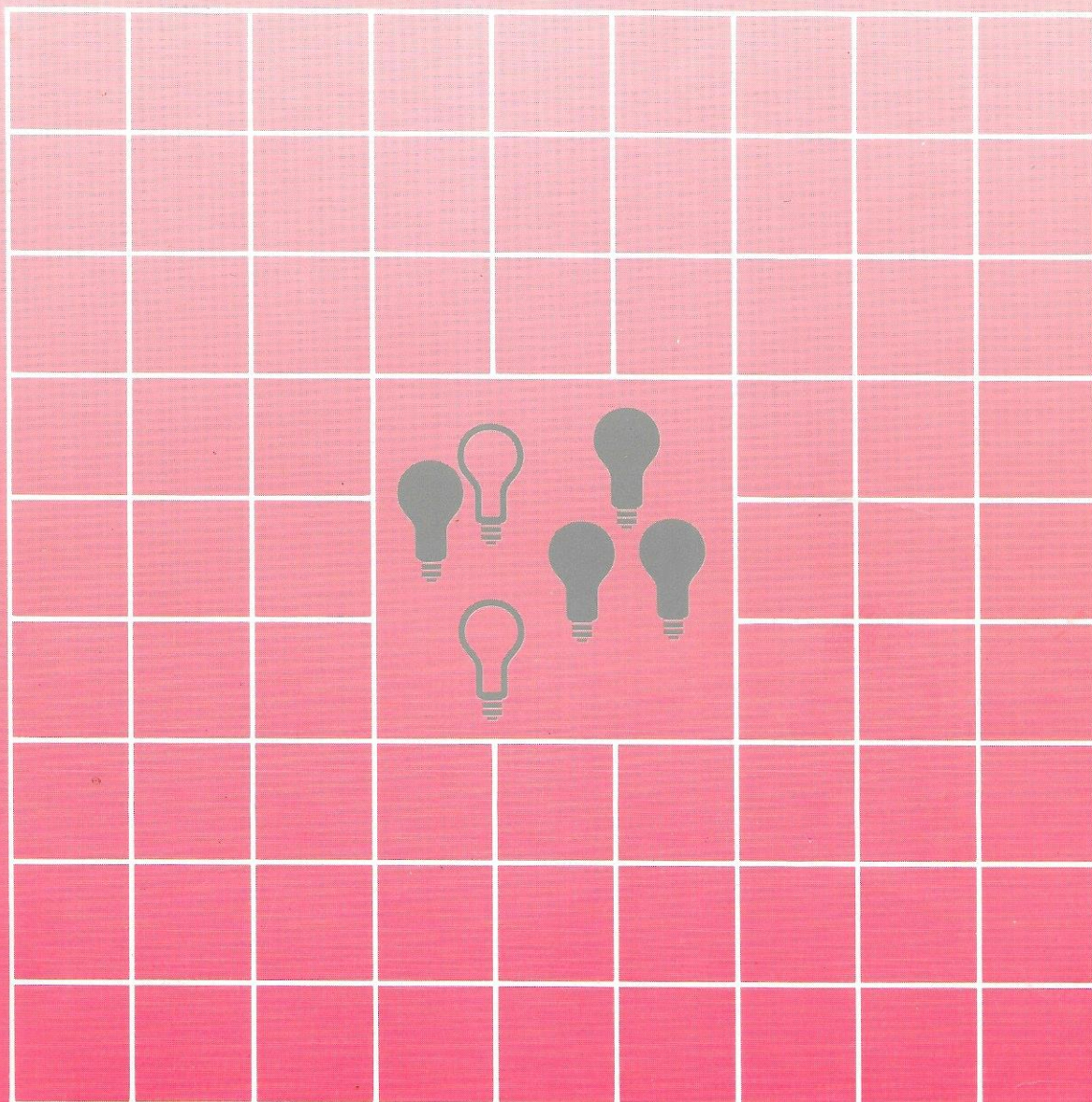
Genetics





Unit 13

Renewal models

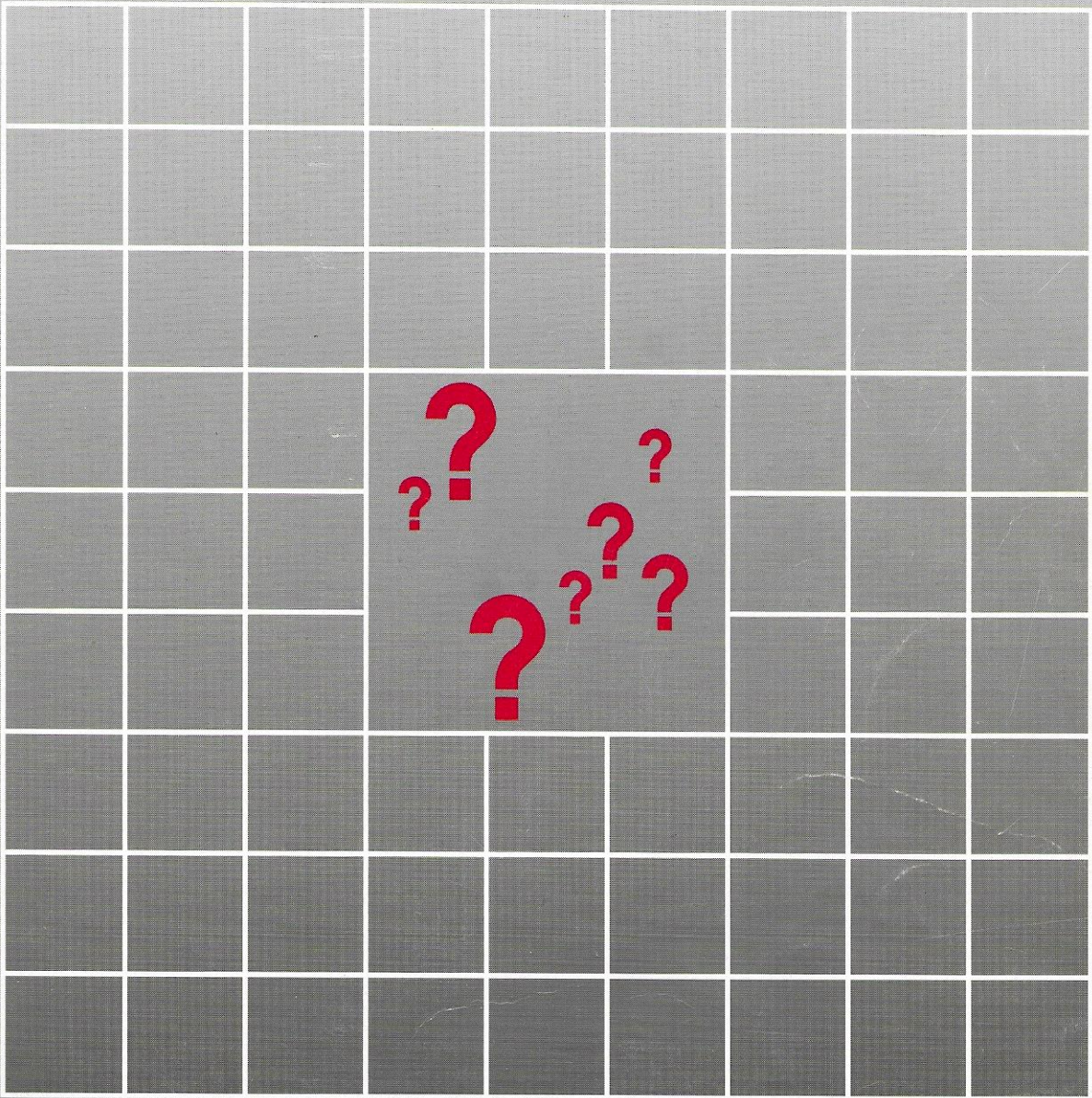


M343 APPLICATIONS OF PROBABILITY



Unit 16

Problems, problems, problems . . .



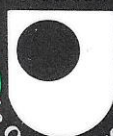
Elementary Statistics Tables

for *all* users of statistical techniques

Henry R. Neave



Set Book



These tables have been carefully prepared for the many users of statistical analysis at an introductory level. The enthusiastic reception accorded to the author's *Statistics Tables* (1978) by specialist statisticians highlighted the need for a briefer set of tables to be tailored to the requirements of students who have to use statistical analysis but with no greater commitment to it than is represented by a basic and often brief introductory course.

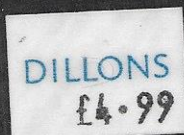
Both the coverage and the presentation of this set of tables have been determined with great care. In contrast with competing sets at this level, the content should match closely the requirements of users, who need have little mathematical background. The book is a positive teaching and learning aid, not just a stark and impenetrable reference item. Most of the tables are accompanied by fully explanatory introductory text and by some examples of use. Each table has been designed and laid out carefully for maximum clarity and ease of use, features which the large page size should also reinforce. There are many new or improved tables, some being much more extensive than in competing books. In view of the increasing recognition of nonparametric tests for their convenience, ease of use and wide application, the tables covering these tests should prove especially valuable.

The tables should serve the needs of all users of elementary techniques of statistical analysis, from engineers and technicians to geographers and social scientists. All students taking first courses in statistical analysis will find them an invaluable aid.

Dr H. R. Neave, also author of *Statistics Tables* (1978), is a lecturer in the mathematics department at Nottingham University. He teaches statistics to undergraduate and post-graduate students, and has also been involved in extra-mural teaching to groups of sixth formers and others. He has a particular interest in nonparametric methods of statistical analysis and has also taught courses in operations research. He has worked in North America on two occasions: as assistant professor at the University of Wisconsin in 1967-8 and as visiting Research Fellow at McGill University in 1970.



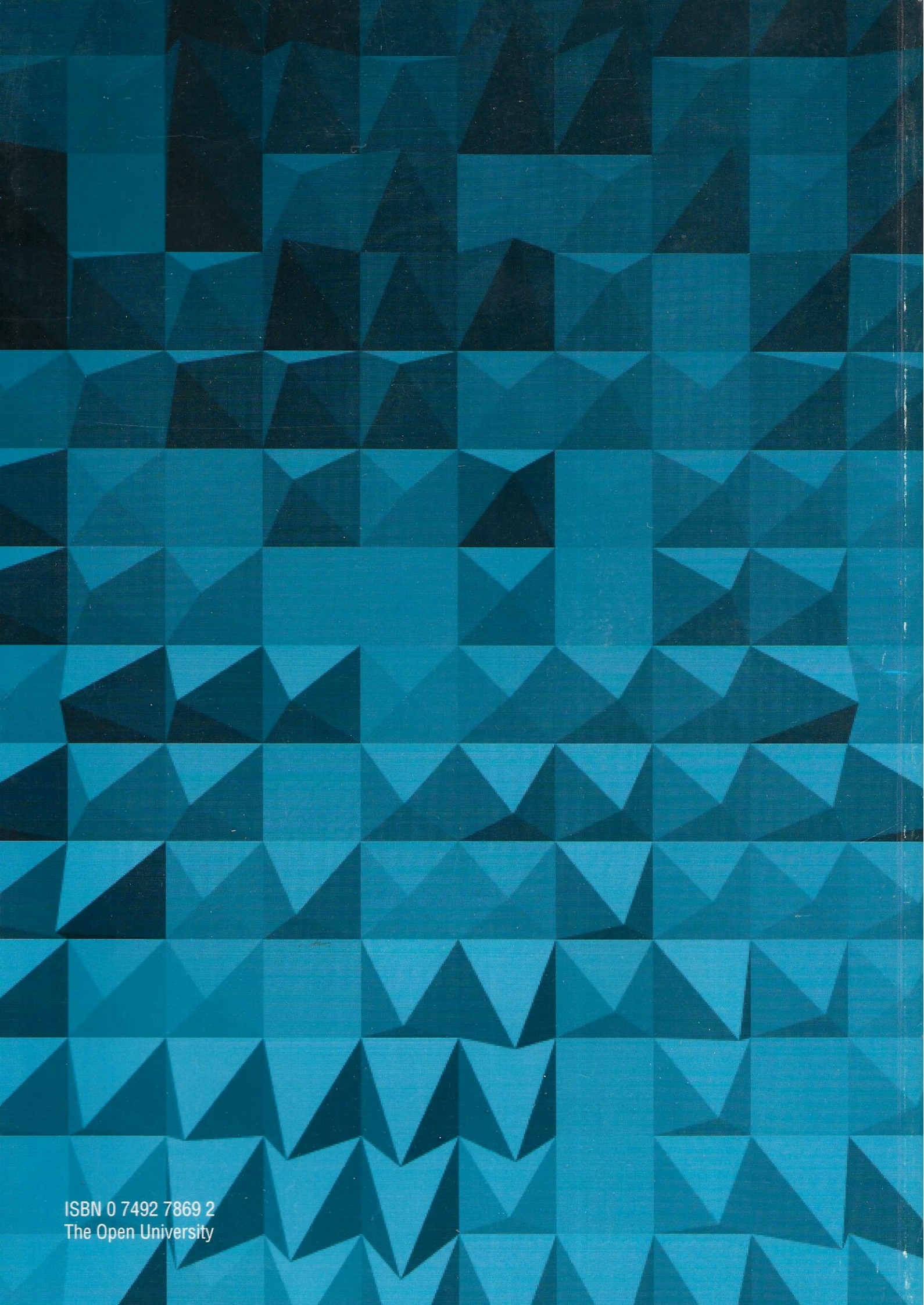
11 New Fetter Lane
London EC4P 4EE



M346
Mathematics and Computing
A third level course



Linear Statistical Modelling

The background of the entire page is a complex, repeating geometric pattern. It consists of numerous triangles of various sizes and shades of blue and black, arranged in a way that creates a three-dimensional, crystalline effect. The triangles are interlocked, forming larger, irregular shapes that resemble a low-poly mesh or a stylized, abstract landscape. The colors range from deep navy blue to a lighter, almost white blue, with black used for the most prominent, sharp edges and points.

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M373 I.1



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I.1

Introduction to iterative methods

Optimization

M373 I.2



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I.2

Systems of linear equations

Optimization

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I.3

Ill-conditioning and induced instability

Optimization

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I.4

Systems of non-linear equations

Optimization

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I.5

Mathematical modelling

Optimization

M373 II.1



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II.1

Linear programming – the basic ideas

Optimization

M373 II.2



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II.2

Linear programming – the two-phase simplex method

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II.3

Integer programming

Optimization

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II.4

Applications of linear and integer programming

Optimization

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III.1

Minimizing a function of one or two variables

Optimization

M373 III.2



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III.2

Unconstrained non-linear optimization

Optimization

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III.3

Constrained non-linear optimization

Optimization

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III.4

Optimization problem-solving

Optimization

M373 CB



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Computing Booklet

Optimization

Mathcad 2001 : toolbar buttons and keyboard short-cuts

• Variables

	Toolbar	Button	Keyboard short-cut
Define (assign a value to)	 	Calculator or Evaluation 	: (colon)
Range variable		Vector and Matrix 	; (semicolon)
Subscripted variable		Vector and Matrix 	[(left square bracket)

• Operators and symbols

Evaluate expression	 	Calculator or Evaluation 	=
Absolute value		Calculator 	(given by [Shift]\)
Square root		Calculator 	\ (back slash)
π (pi)	 	Calculator or Greek 	[Ctrl][Shift]p
Equal to (Boolean equals)		Boolean 	[Ctrl]=

Other useful keys – [F9] for manual calculation, [Ctrl]r to refresh (redraw) the screen, [F1] for help and . (full stop) to create a literal subscript in a variable name.

• Graphs

		Graph toolbar	
X-Y Plot		or type @ ([Shift]')	Zoom  Trace 

• Vectors and matrices

		Vector and Matrix toolbar	
Create a matrix or vector		or type [Ctrl]m	
Determinant (or magnitude)		([Shift]\)	Transpose  [Ctrl]1

• Differentiation, integration, summation and iterated products

		Calculus toolbar	
Derivative		or type ?	Higher derivative  [Ctrl]?
Indefinite integral		[Ctrl]i	Definite integral  & ([Shift]7)
Summation		[Ctrl][Shift]4	Iterated product  [Ctrl][Shift]3

• Symbolic calculations

		Symbolic Keyword toolbar	
Symbolic evaluation		or type [Ctrl].	
Symbolic keywords, for example – simplify		[Ctrl][Shift].simplify	

(There are also symbolic keywords for expand, solve, float, substitute, complex, factor.)

M373 HB



Mathematics and Computing
A level 3 course

Handbook

Optimization

COMPUTABILITY AND LOGIC

Third edition

George S. Boolos and
Richard C. Jeffrey



Open University Set Book

CAMBRIDGE
UNIVERSITY PRESS

This third edition of *Computability and Logic* has been corrected and contains thoroughly revised versions of the chapters on Ramsey and provability, with new exercises provided for three other chapters. There are also two new chapters dealing with undecidable sentences and on the non-existence of non-standard recursive models of Z .

"... gives an excellent coverage of the fundamental theoretical results about logic involving computability, undecidability, axiomatization, definability, incompleteness, etc."

American Math Monthly

"... particularly appropriate for graduate and advanced undergraduate students in philosophy. . . . The book is written in a clear and pleasing style and avoids pedantry. . . . It should be an excellent text for its intended audience."

Mathematical Reviews

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ISBN 0-521-38923-2



9 780521 389235

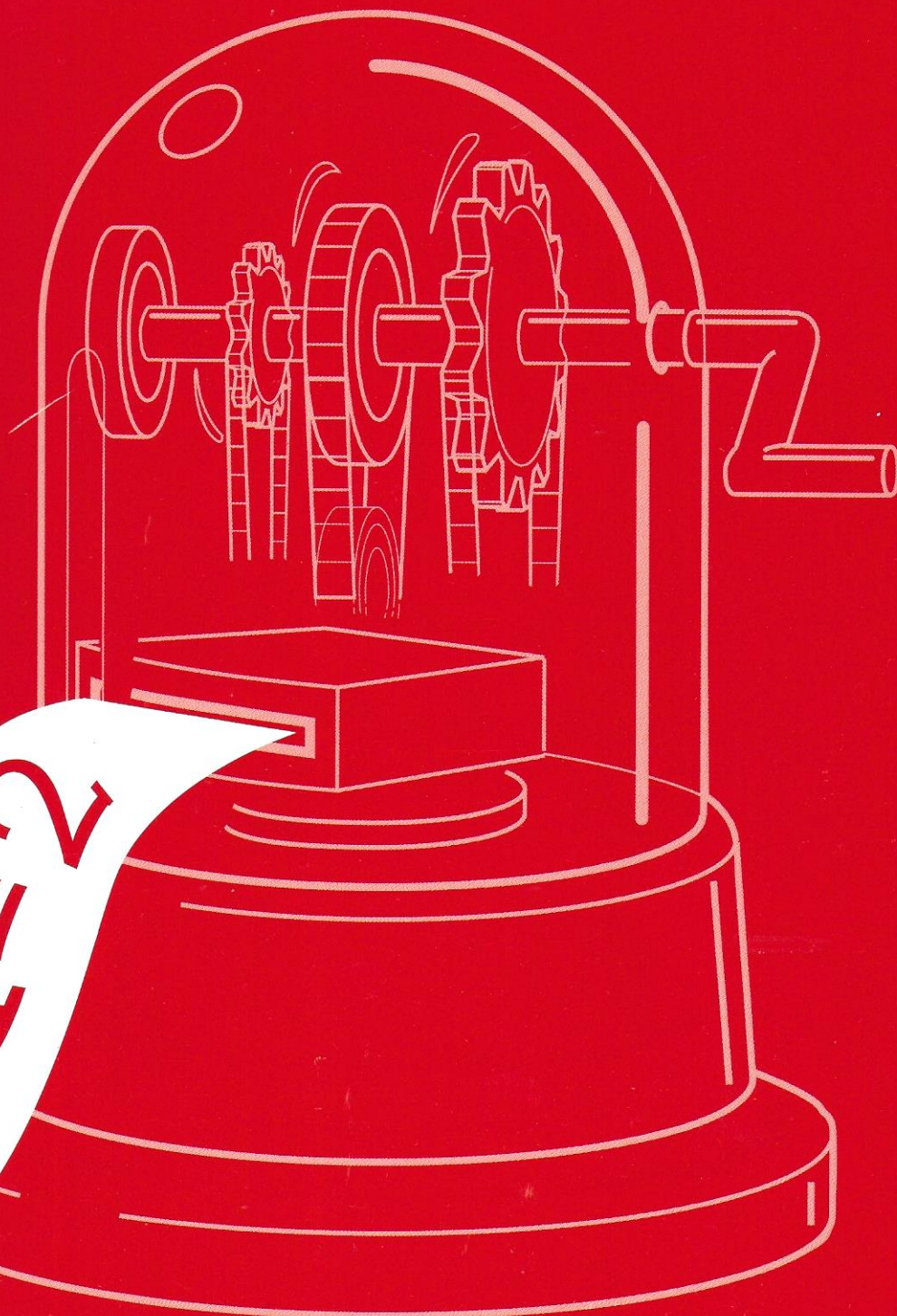
Mathematical Logic

Unit 1

Computability

This computation doesn't halt
 $t(n, m)$


$$1 + 1 = 2$$

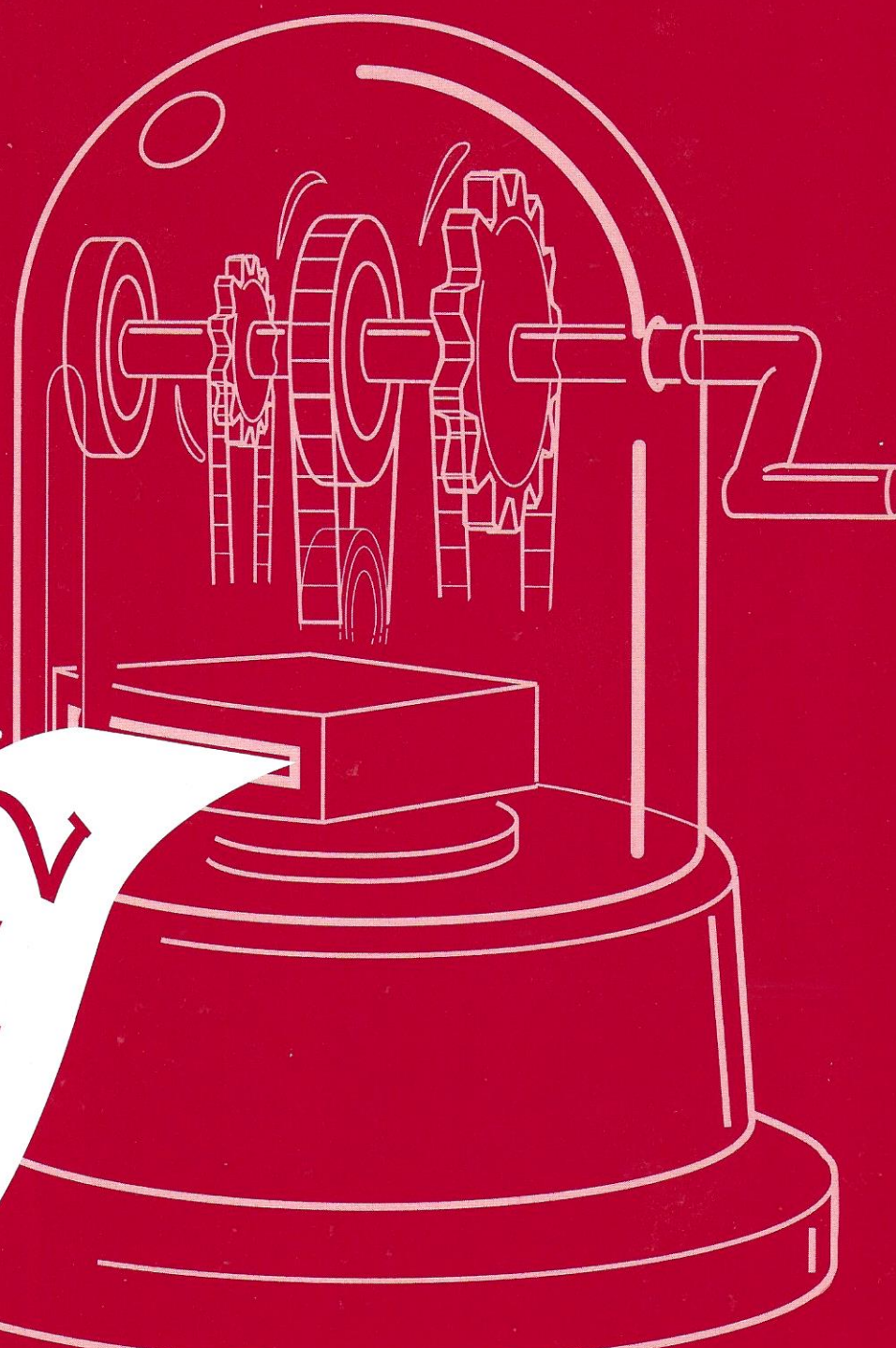


Mathematical Logic

Unit 2

Primitive Recursive Functions

This computation doesn't halt
 $t(n, m)$


$$1 + 1 = 2$$



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M381 Number Theory and Mathematical Logic
Mathematics and Computing
A third level course

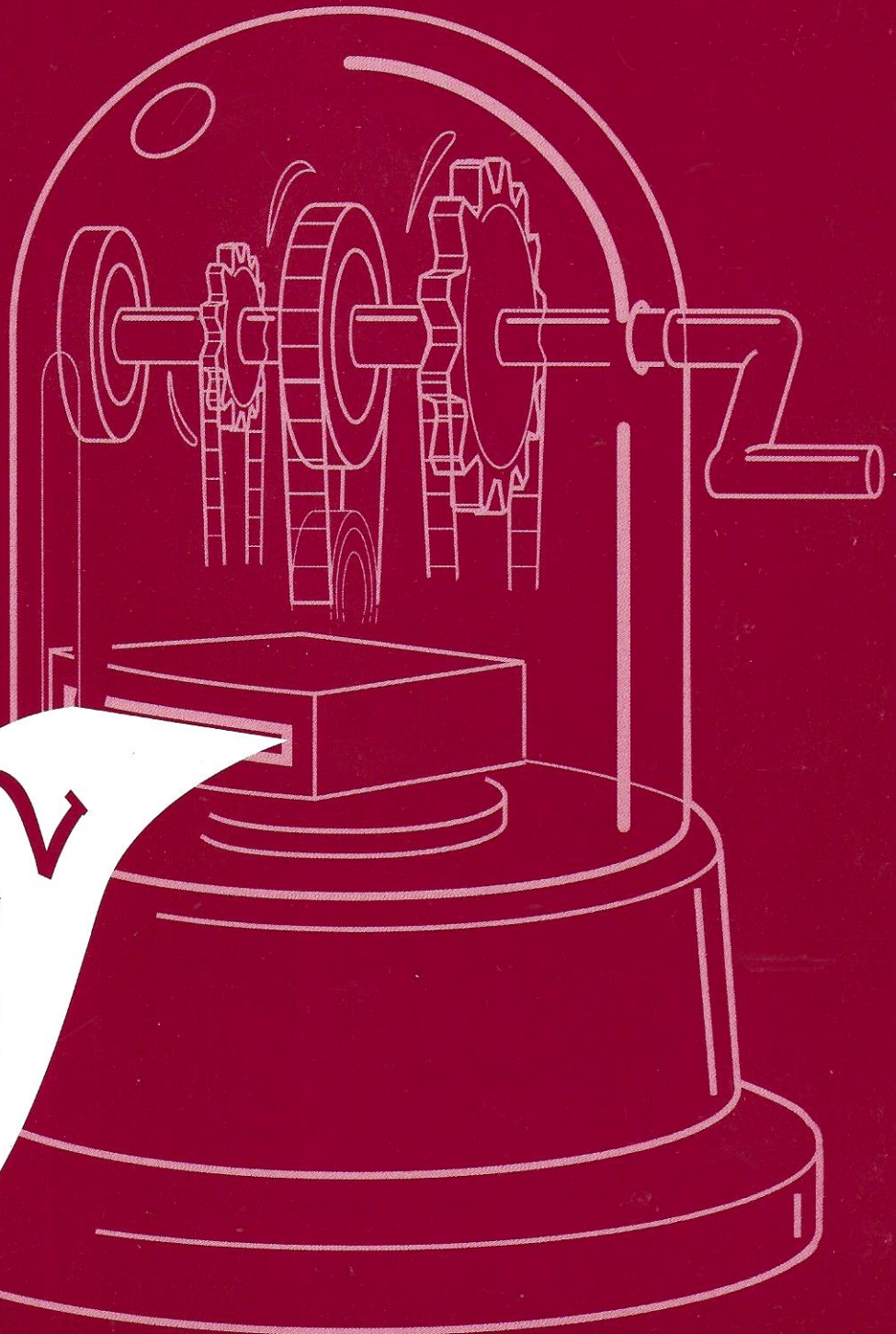
Mathematical Logic

Unit 3

Church's Thesis

This computation doesn't halt
(n, m)

$$1 + 1 = 2$$

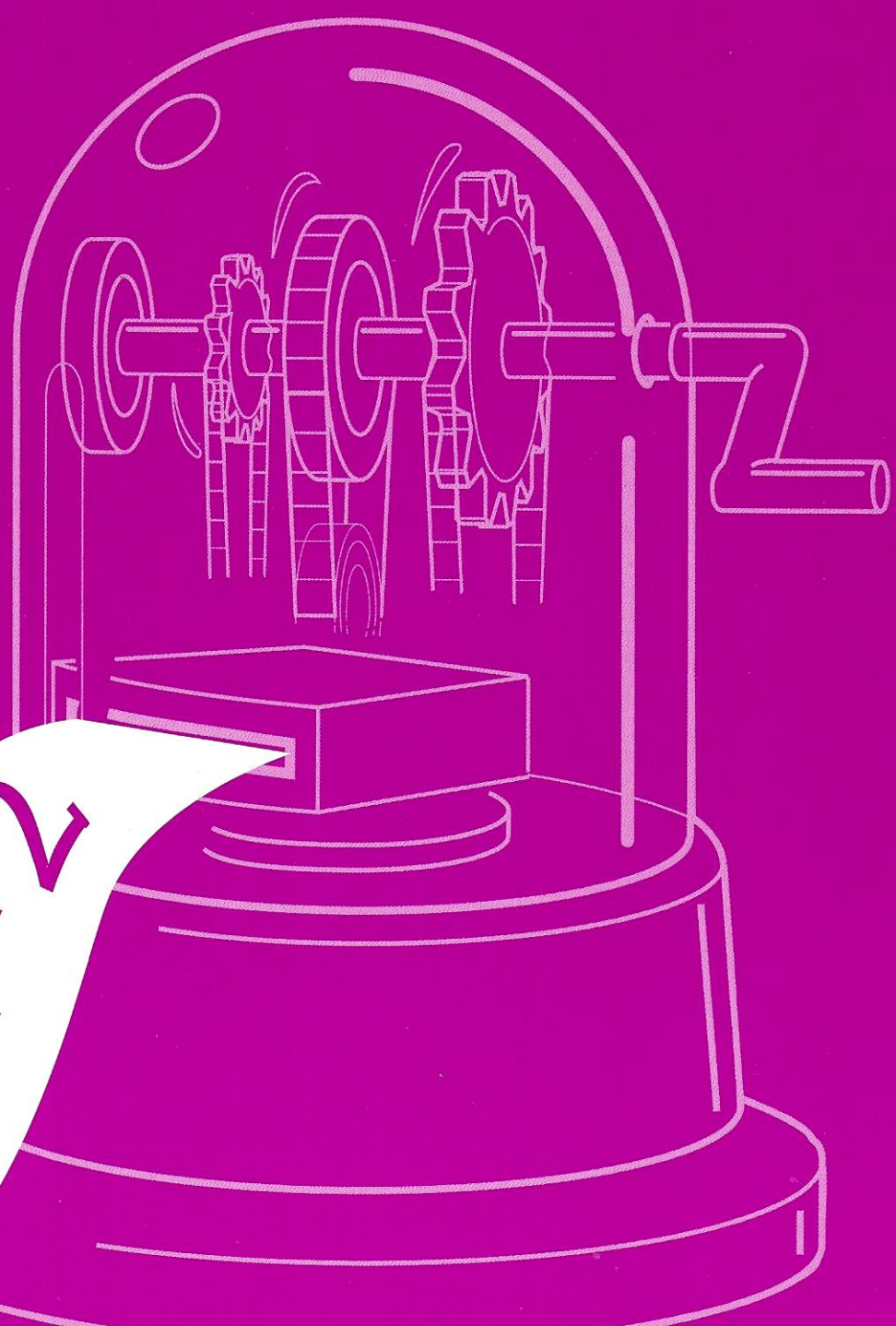


Mathematical Logic

Unit 4

Formal Systems

This computation doesn't halt
 (n, m)

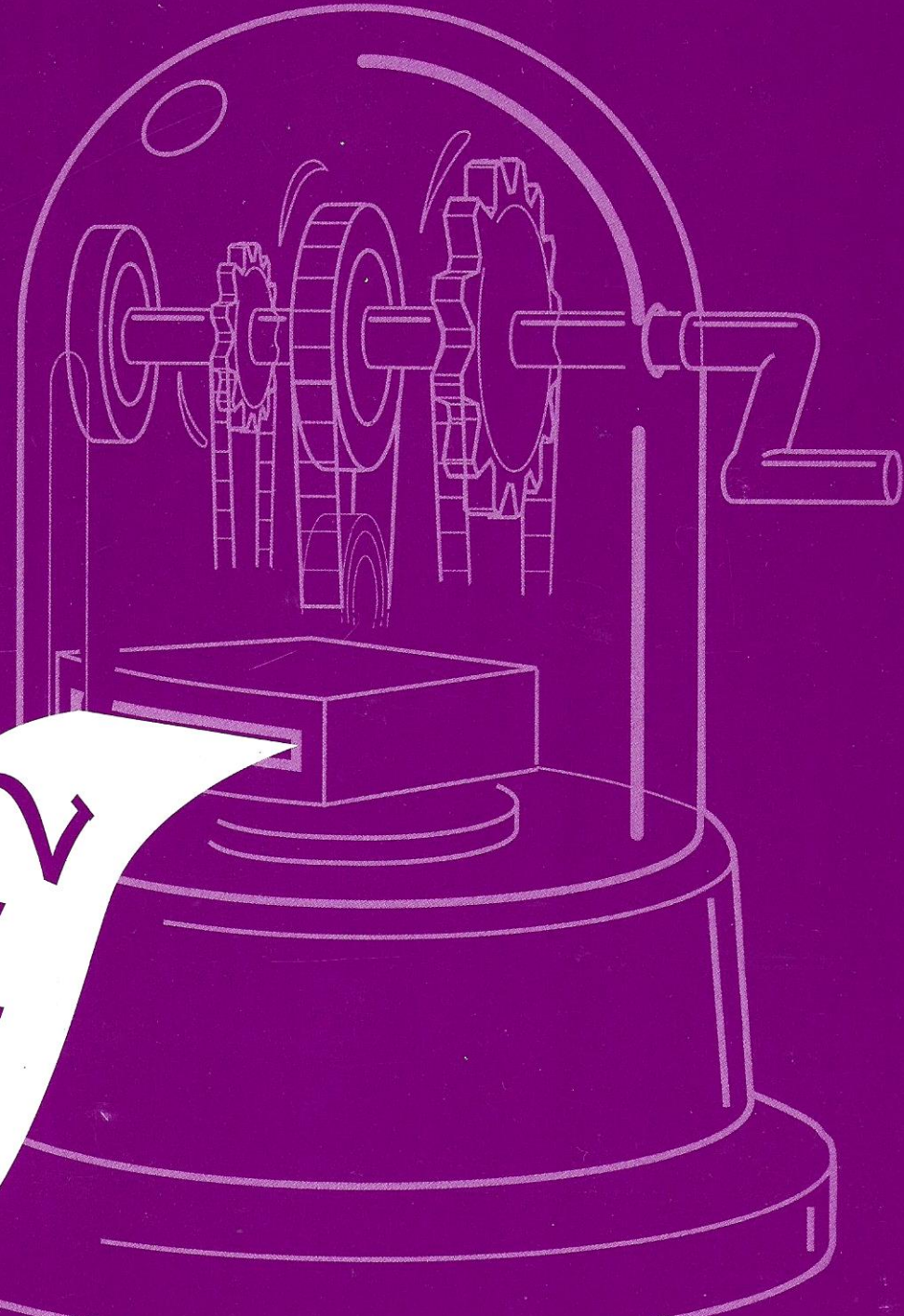


$1 + 1 = 2$

Mathematical Logic

Unit 5 Formal Proof

This computer doesn't halt
(n, m)



$1 + 1 = 2$

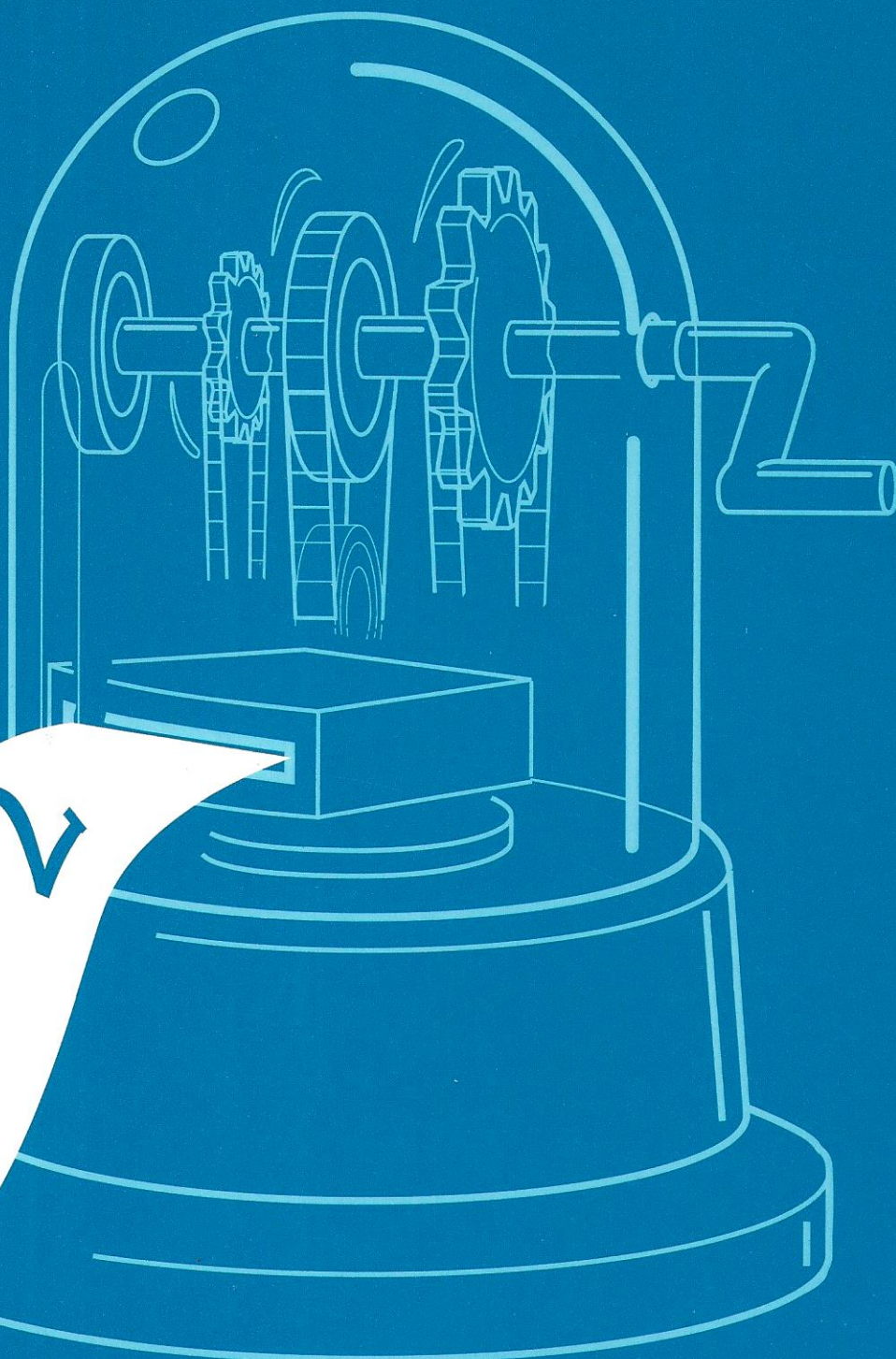
Mathematical Logic

Unit 6

Formal Number Theory I

This computation doesn't halt
 $t(n, m)$

$$1 + 1 = 2$$



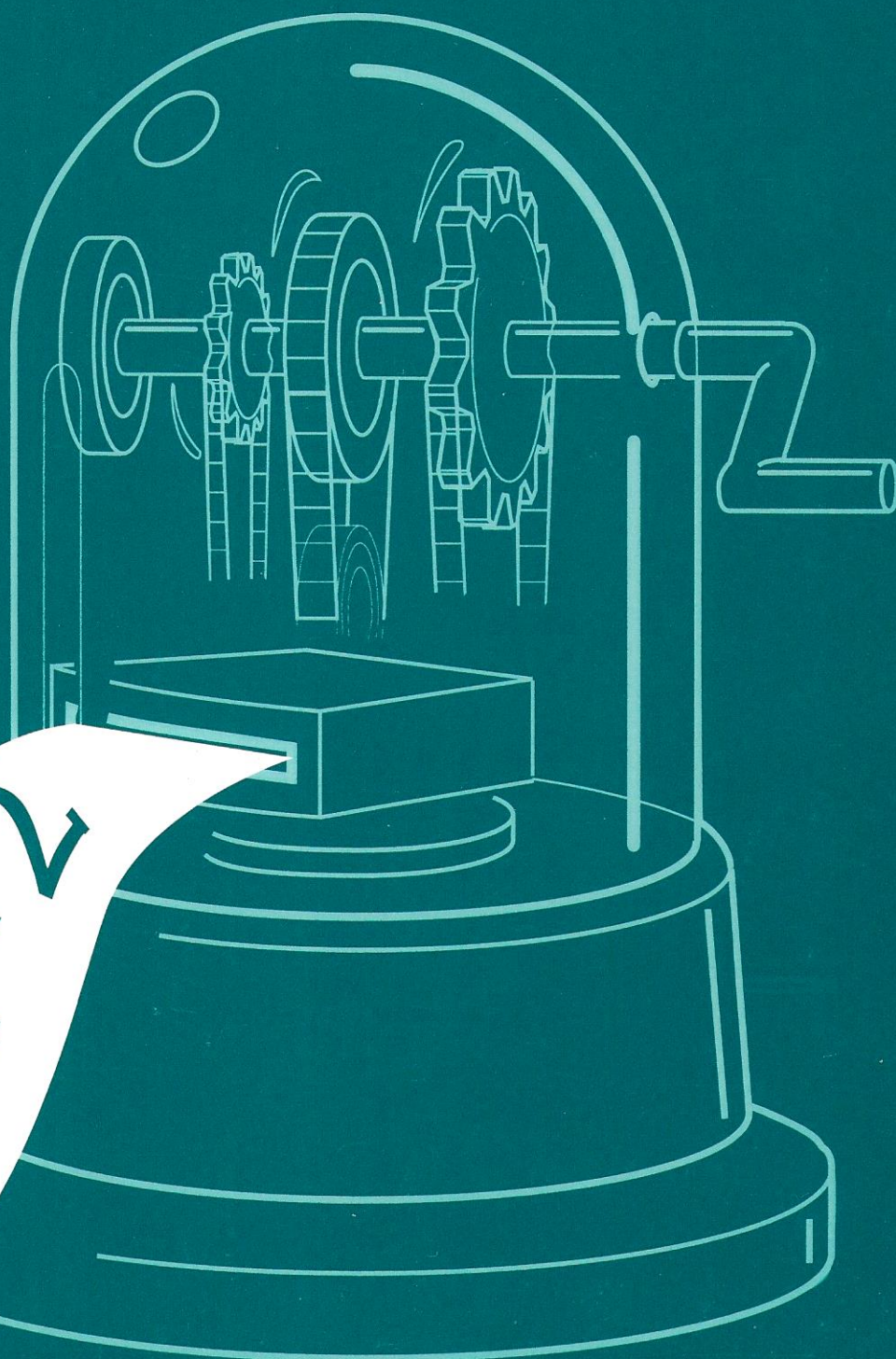
Mathematical Logic

Unit 7

Formal Number Theory II

This computation doesn't halt
 $\neg \text{halt}(n, m)$


$$1 + 1 = 2$$



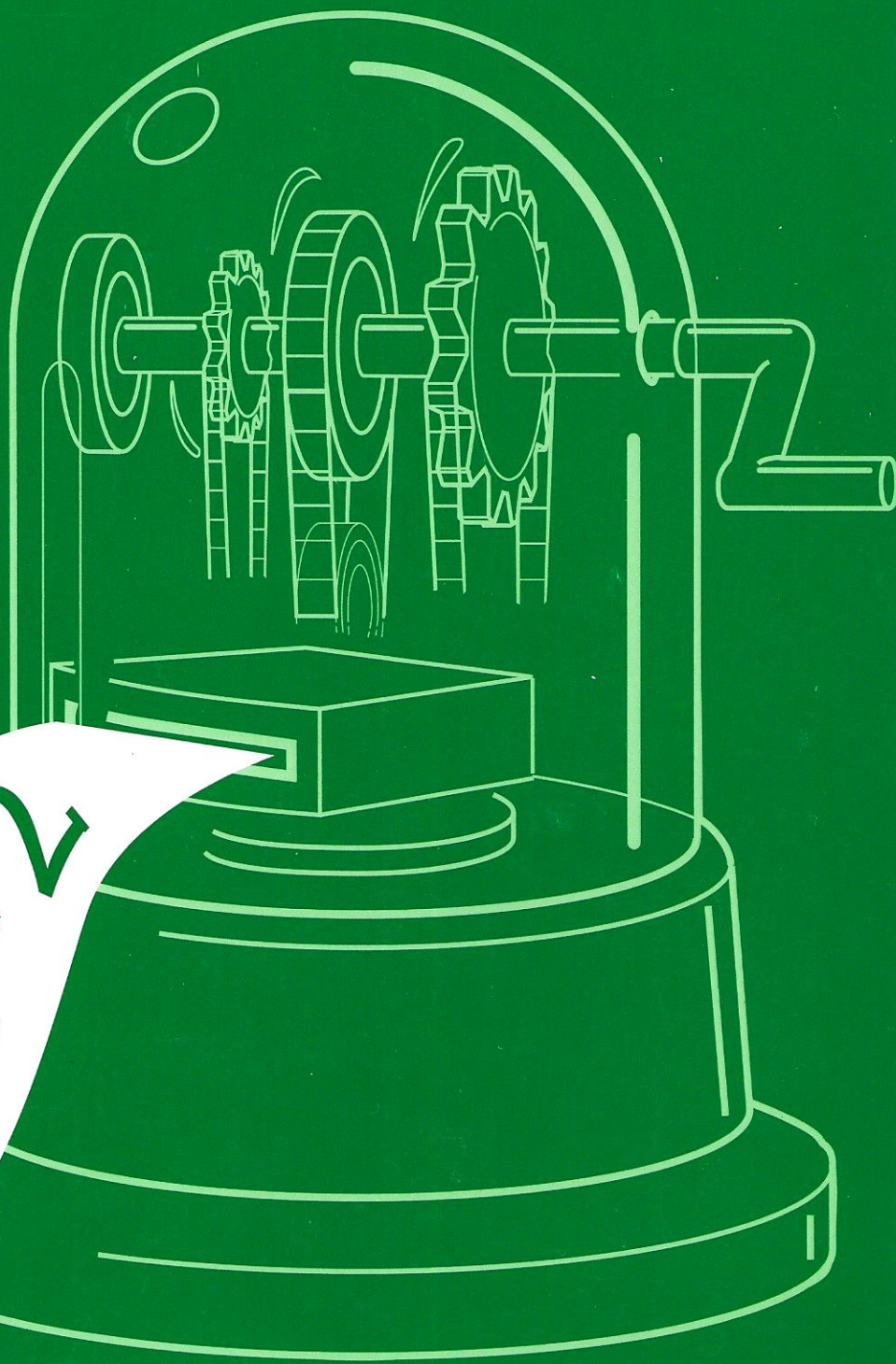
Mathematical Logic

Unit 8

Gödel's Incompleteness Theorems

This computer program doesn't halt
(n, m)


$$1+1=2$$



Number Theory

Unit 1 Foundations

97		96		95		94		93		92		91		90
98	71		70		69		68		67		66		89	
99	72			53		52		51		50		65		88
100	73		54		43		42		49		64		87	
101	74		55		44		41		48		63		86	
102	75		56		45		46		47		62		85	
103	76		57		58		59		60		61		84	
104	77		78		79		80		81		82		83	

Number Theory

Unit 2

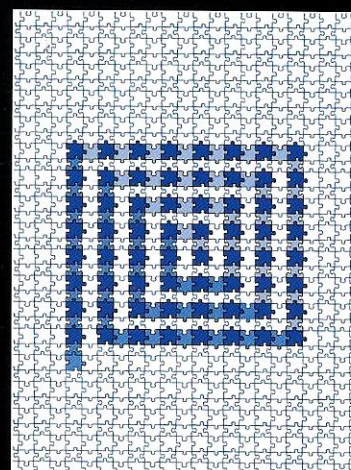
Prime Numbers

97		96		95		94		93		92		91		90
98	71		70		69		68		67		66		89	
99	72			53		52		51		50		65		88
100	73		54		43		42		49		64		87	
101	74		55		44		41		48		63		86	
102	75		56		45		46		47		62		85	
103	76		57		58		59		60		61		84	
104	77		78		79		80		81		82		83	



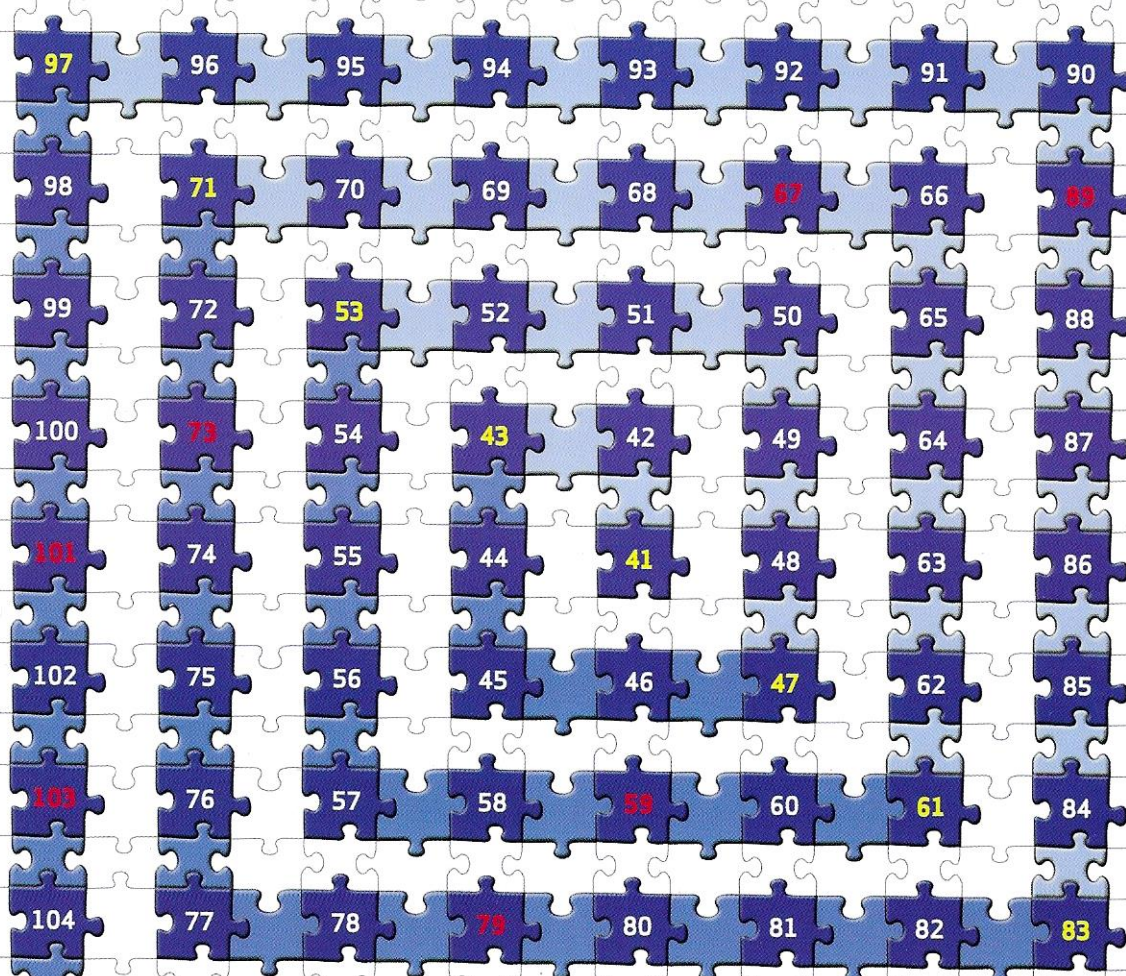
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M381 Number Theory and
Mathematical Logic



Number Theory **Unit 3**

Congruence





Number Theory

Unit 4 Fermat's and Wilson's Theorems

97		96		95		94		93		92		91		90
98	71		70		69		68		67		66		89	
99	72		53		52		51		50		65		88	
100	73		54		43		42		49		64		87	
101	74		55		44		41		48		63		86	
102	75		56		45		46		47		62		85	
103	76		57		58		59		60		61		84	
104	77		78		79		80		81		82		83	

Number Theory

Unit 5 Multiplicative Functions

97		96		95		94		93		92		91		90
98	71		70		69		68		67		66		89	
99	72	53		52		51		50		65		88		
100	73	54	43		42		49		64		87			
101	74	55	44	41	48		63		86					
102	75	56	45	46	47		62		85					
103	76	57	58	59	60	61	84							
104	77	78	79	80	81	82	83							

Number Theory

Unit 6

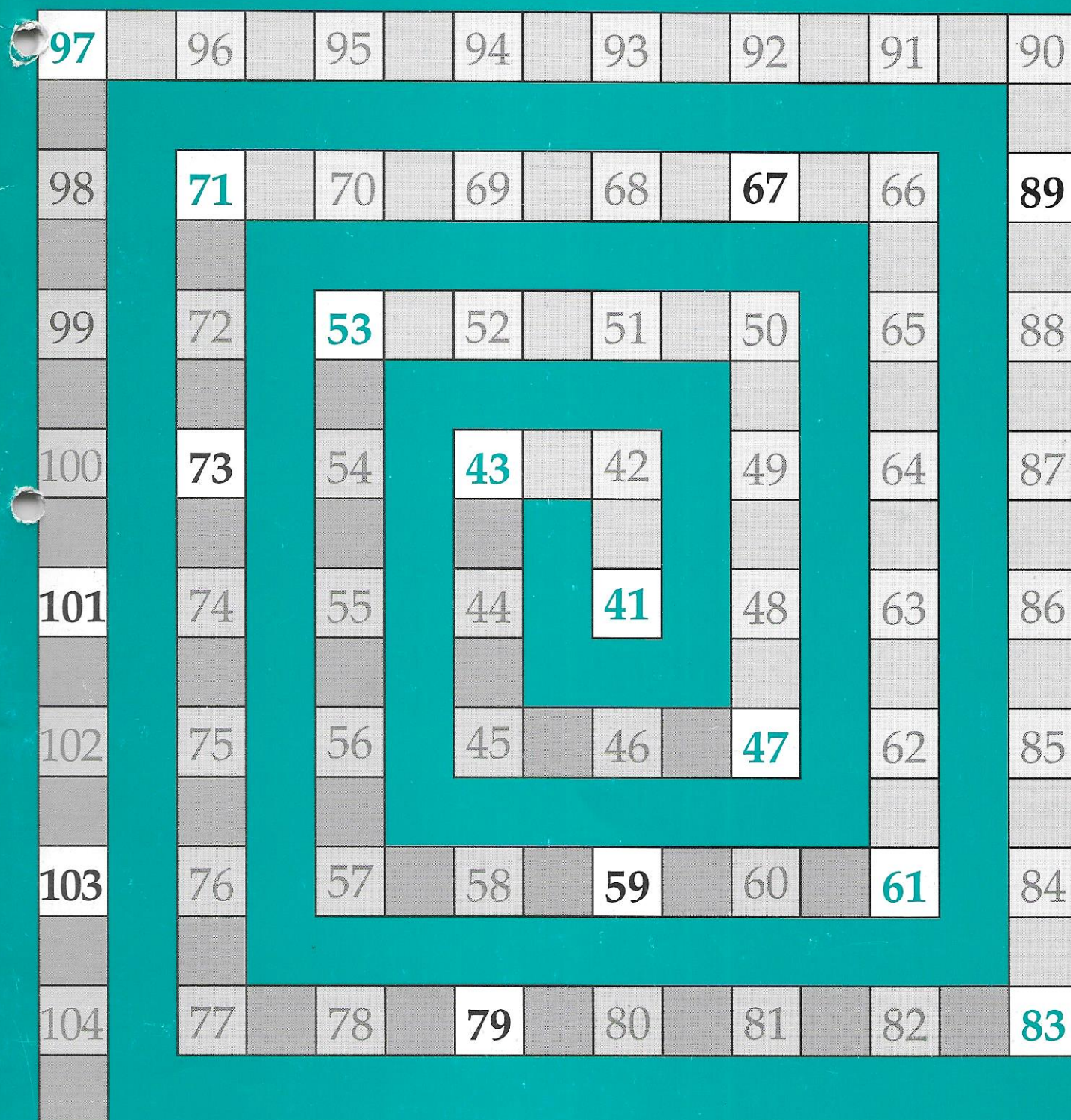
Quadratic Reciprocity

97		96		95		94		93		92		91		90
98	71		70		69		68		67		66		89	
99	72		53		52		51		50		65		88	
100	73		54		43		42		49		64		87	
101	74		55		44		41		48		63		86	
102	75		56		45		46		47		62		85	
103	76		57		58		59		60		61		84	
104	77		78		79		80		81		82		83	

Number Theory

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Unit 7 Continued Fractions



Number Theory

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Unit 8 Diophantine Equations

97		96		95		94		93		92		91		90
98	71		70		69		68		67		66		89	
99	72	53		52		51		50		65		88		
100	73	54	43		42		49		64		87			
101	74	55	44		41		48		63		86			
102	75	56	45		46		47		62		85			
103	76	57	58		59		60		61		84			
104	77	78	79		80		81		82		83			

Mathematics/Science:
An Interfaculty Third Level Course

MS323
BLOCK I
UNIT 1

*INTRODUCTION TO
NON-LINEAR DYNAMICS*

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An Interfaculty Third Level Course

MS323
BLOCK I
UNITS 2–4

*INTRODUCTION TO
NON-LINEAR DYNAMICS*

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BLOCK II
UNITS 5–7

*INTRODUCTION TO
NON-LINEAR DYNAMICS*

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MS323
BLOCK III
UNITS 8–10

***INTRODUCTION TO
NON-LINEAR DYNAMICS***

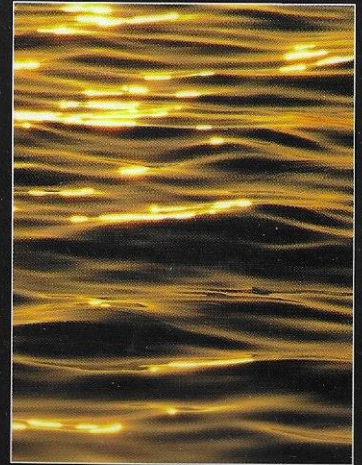
Mathematics/Science:
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MS323
BLOCK IV
UNITS 11–13

*INTRODUCTION TO
NON-LINEAR DYNAMICS*

MS324 Waves, diffusion and
variational principles

0



Block 0 Preparatory material



MS324 Waves, diffusion and variational principles

Block 0

► Preparatory material

Block I

Waves

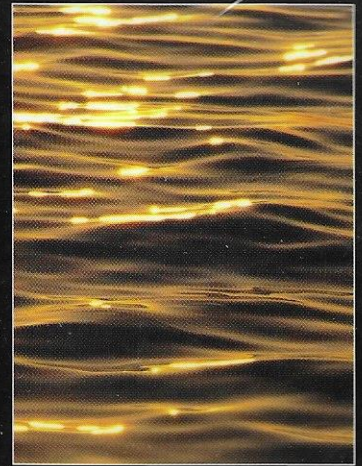
Block II

Random walks and diffusion

Block III

Variational principles

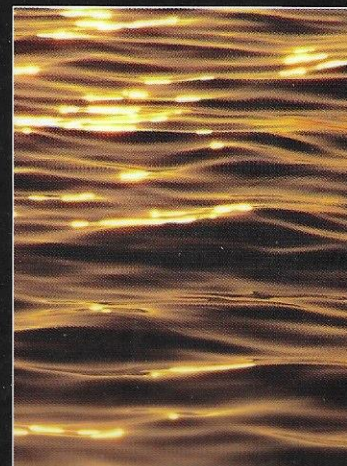
MS324 Waves, diffusion and
variational principles



Block I Waves



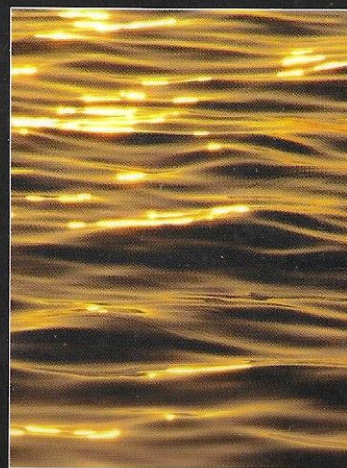
MS324 Waves, diffusion and
variational principles



Block II Random walks and diffusion

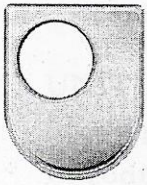


MS324 Waves, diffusion and
variational principles



Block III Variational principles





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MS325 Computer algebra, chaos and simulations

**EXAMPLE
ONLY**

Block A

Computer algebra

UNIT 2 More Maple

Study guide

Sections 2.1–2.4 each build upon the results of the previous sections and should be studied in numerical order. Sections 2.5 and 2.6 are largely independent and can be studied in either order after Section 2.4.

Access to a computer is required to study this unit.

Introduction

In *Unit 1* you met some basic Maple commands. Generally, these commands converted an expression to another expression by some process (e.g. simplifying, differentiating, integrating, etc.). In this unit we consider commands which create and manipulate more structured objects such as lists, sets, vectors and matrices. This will increase greatly the range of problems that you will be able to solve in Maple.

The unit starts by looking at how functions can be defined in Maple. Maple functions are needed when an expression needs to be evaluated repeatedly – such as manipulating lists of data. Maple functions are very closely related to Maple procedures, which you will meet in *Unit 3*.

Section 2.2 looks at some of the ways that Maple can store collections of things (data, variables, equations, etc.) and discusses some methods of manipulating them.

Section 2.3 introduces the concept of a power series and explains how Maple can truncate such a series to a polynomial.

Unit 1 introduced line graphs produced using the `plot` command. In Section 2.4 we describe some of the other types of graph that Maple can produce, such as the 3D-surface plot shown in Figure 2.1.

Some of the more specialized graphics commands are contained within a ‘Maple package’. Maple packages are an important feature of Maple in that they contain commands that extend Maple to address problems in particular problem areas. Maple has many packages (see `?index/packages` for a list), but in this course we concentrate on the core of Maple and mention only a few packages. Subsection 2.4.3 describes how to access and use the commands that reside in packages.

The first four sections of this unit contain basic Maple commands that are used in many application areas. Section 2.5 examines some of the tools that are available in Maple for linear algebra calculations. These tools are applied to solve problems such as classifying the stationary points of surfaces like the one shown in Figure 2.1.

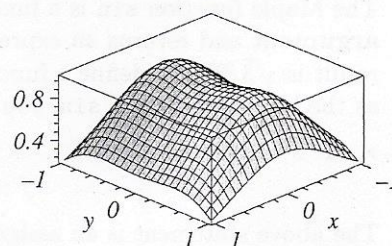


Figure 2.1 `plot3d` output

UNIT 3 Programming

Study guide

This unit builds on the results of *Unit 1* and *Unit 2*, and the three sections should be studied in numerical order. Apart from reading the text and understanding the examples, it is crucial that you work through the exercises, as solving these is essential to learning the material.

You will require access to a computer to study this unit.

Introduction

In this unit we explore programming in Maple. A computer program is a collection of instructions (sometimes called an algorithm), that describe a task and enable it to be carried out by a computer. Without programs, computers are completely useless. All of the software on your PC, including the operating system, is a computer program, and almost every modern electronic device, from washing machines to aircraft, is controlled by such programs.

Computer programs are written in a 'computer programming language', of which there are thousands: C, C++, BASIC, FORTRAN and Maple are just a few. Maple is a very specialized programming language, important for scientific and mathematical applications. It and similar computer algebra languages are now routinely used in mathematical and scientific research and education.

In the previous two units, you learned how to use a number of Maple commands and how to combine them to solve a variety of mathematical problems. This approach is particularly useful when the mathematical problems are relatively simple, or for exploring different approaches to solving unfamiliar problems. However, some tasks require the use of relatively few commands very many times. When it becomes excessively laborious to input all of these commands by hand, it is necessary to have some method of automating their execution. This is achieved by writing a computer program.

There are three key programming constructs that Maple provides which are common to most programming languages, namely, loops, conditionals and procedures.

Loops enable the execution of many lines of Maple code repeatedly and sequentially. Simple loops are explored in Section 3.1, where several examples are presented in order to illustrate the sort of problems to which

UNIT 4 Case studies

Study guide

This unit consists of six sections, of roughly equal length. They may be studied in any order (though it makes sense to study Section 4.5 before Section 4.6). They are presented in what is felt to be a sensible order in terms of content and difficulty. Although there is material in each section that does not require a computer, you will need access to a computer at some stage in every section.

Ideally you should have time to study the whole of this unit. However, if time is short then you should try to complete at least four of the sections, including Section 4.6.

Even if you do not study Sections 4.5 and 4.6 in detail, you should introduce yourself to the series option in `dsolve`, which is introduced in Section 4.5, and to `odeplot`, which is described in Section 4.6.

Section 4.6 also uses the concatenation operation `cat` to produce plots with informative titles. Although you will not be required to use `cat` for assessment purposes, you are recommended to study this helpful facility.

Introduction

This unit looks at six case studies that explore different types of problems that can be tackled with Maple. Working through the unit should enable you to consolidate the Maple commands and structures that you learned in *Units 1–3*.

Section 4.1 uses Maple to investigate some properties of special sequences of numbers called *spectra*. You will use your computer to explore patterns that emerge from these sequences, from which conjectures about the patterns can be made. Maple can then be used to verify the conjectures for specific cases.

Section 4.2 looks at the topic of *numerical integration*. You may already be familiar with the *trapezium rule*, which can be used to approximate the value of a definite integral that cannot be found exactly. This section outlines both the trapezium rule and the more general *Newton-Cotes formula*, and shows how Maple can be used with these methods to give good approximations to definite integrals.

In Section 4.3 a method of finding polynomial approximations to functions of a single variable is introduced. Maple can be very useful here, in finding the required polynomial coefficients. The close connection between this

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Block A

Unit 1 Basic commands

Prepared for the Course Team

by Catherine Wilkins

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Block B

Unit 1 Introduction to dynamical systems

Prepared for the Course Team

by Paul J. Upton

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Unit B2 One-Dimensional Maps

Prepared for the Course Team

by Paul J. Upton

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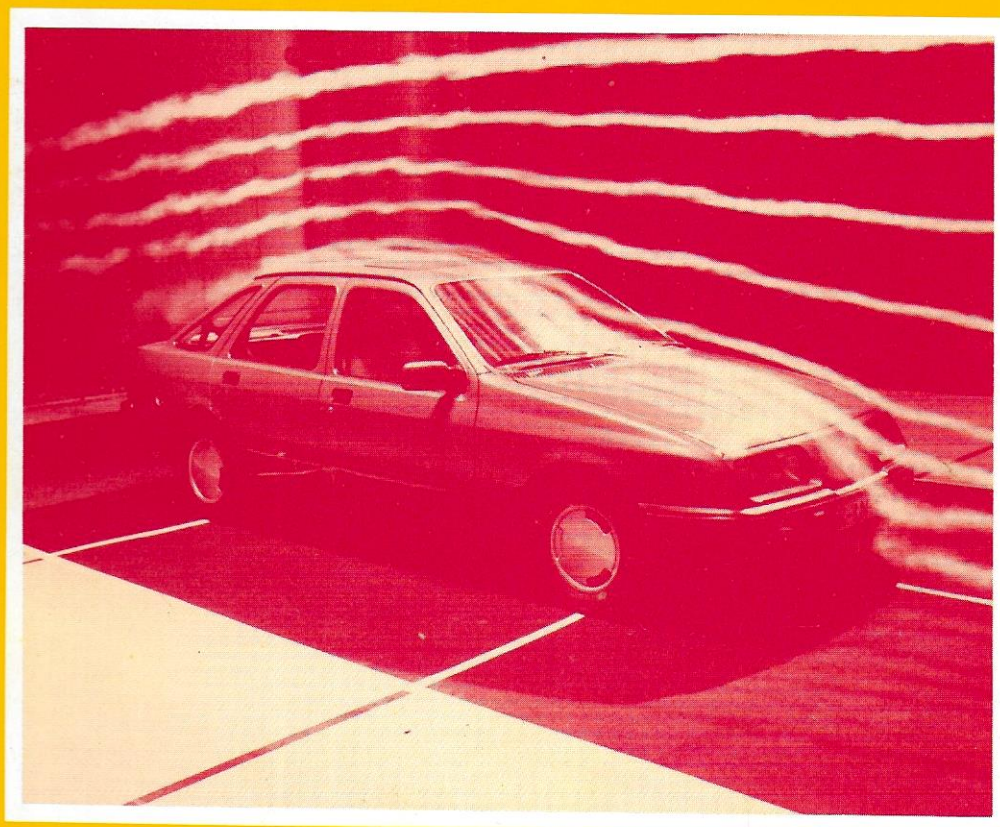
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Mathematical Methods and Fluid Mechanics

Unit 1

Properties of a fluid



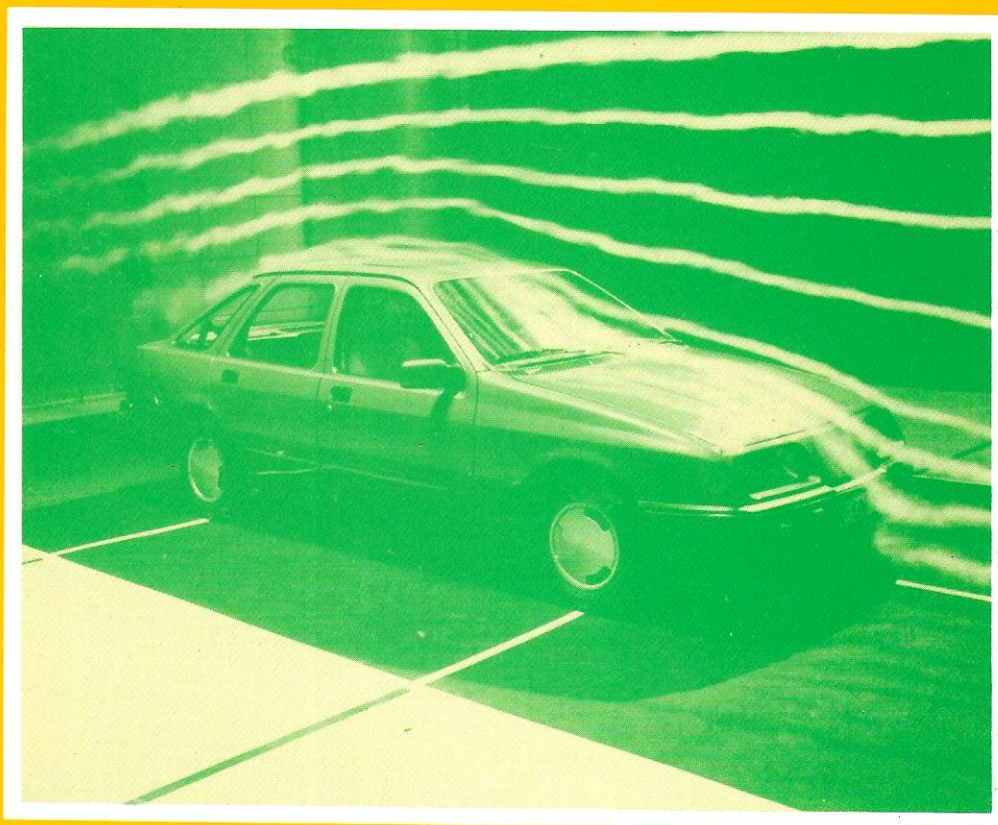
$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



Mathematical Methods and Fluid Mechanics

Unit 2

Boundary-value problems



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



Mathematical Methods and Fluid Mechanics

Unit 3

Part I First-order partial differential equations

Part II Dimensional methods



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$

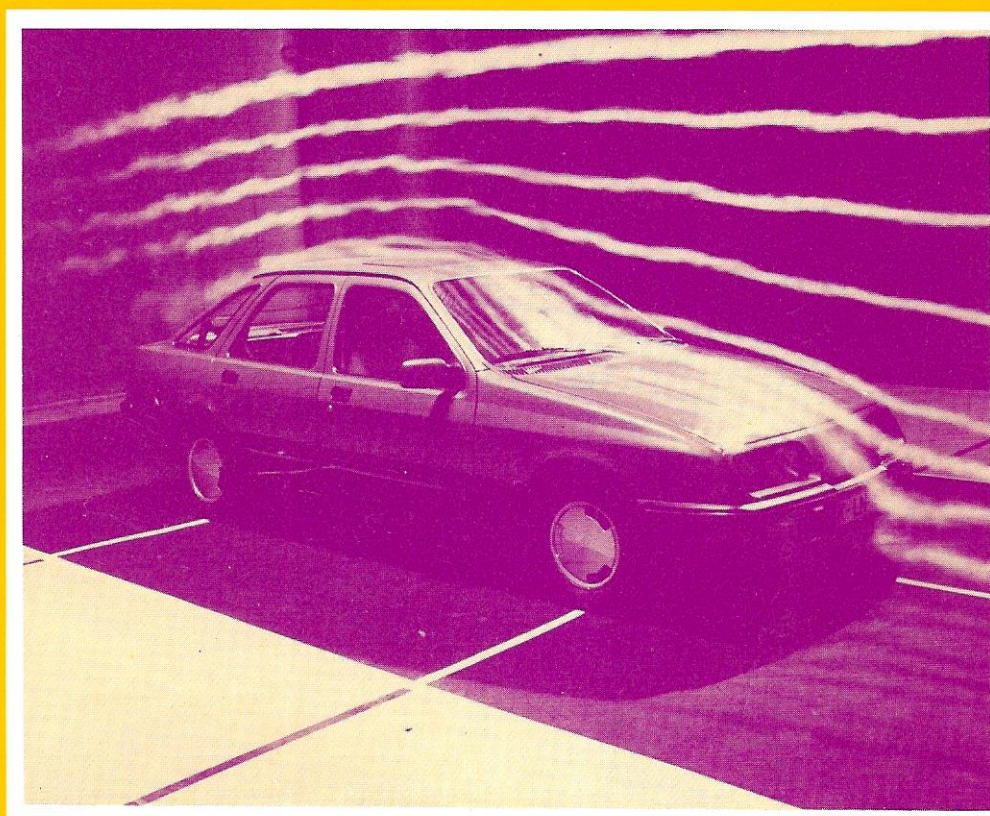
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



Mathematical Methods and Fluid Mechanics

Unit 4

Vector field theory



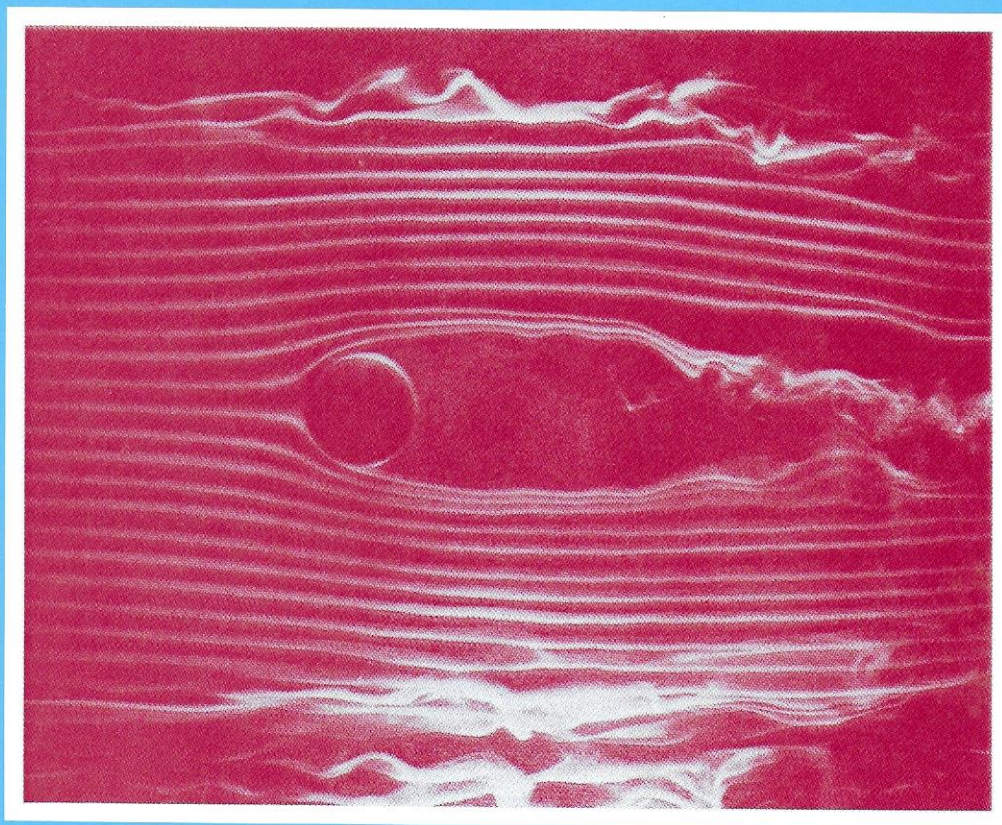
$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



Mathematical Methods and Fluid Mechanics

Unit 5

Kinematics of fluids



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$

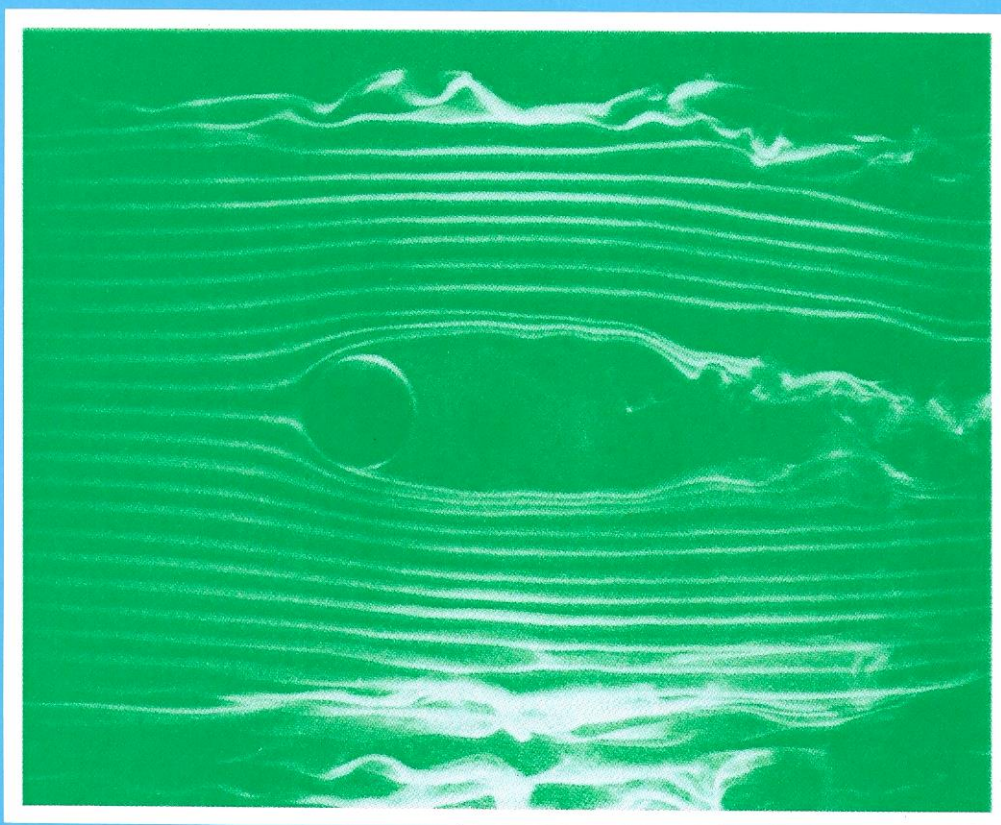
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



Mathematical Methods and Fluid Mechanics

Unit 6

Bernoulli's equation



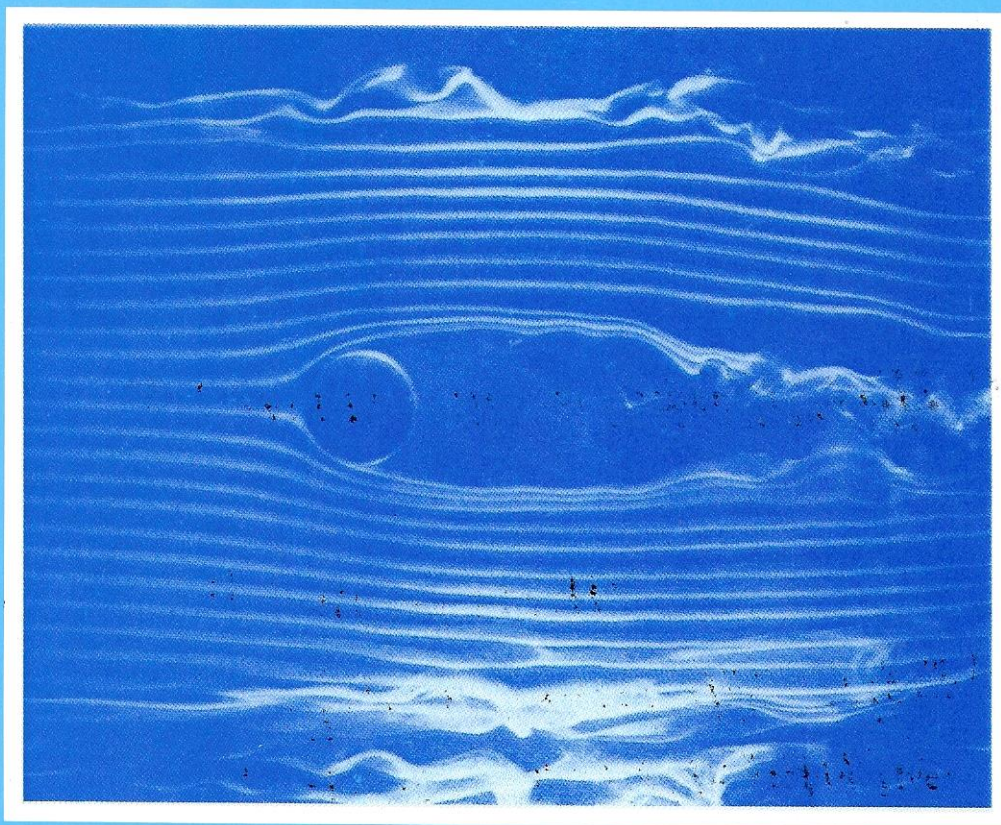
$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



Mathematical Methods and Fluid Mechanics

Unit 7 Vorticity



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$

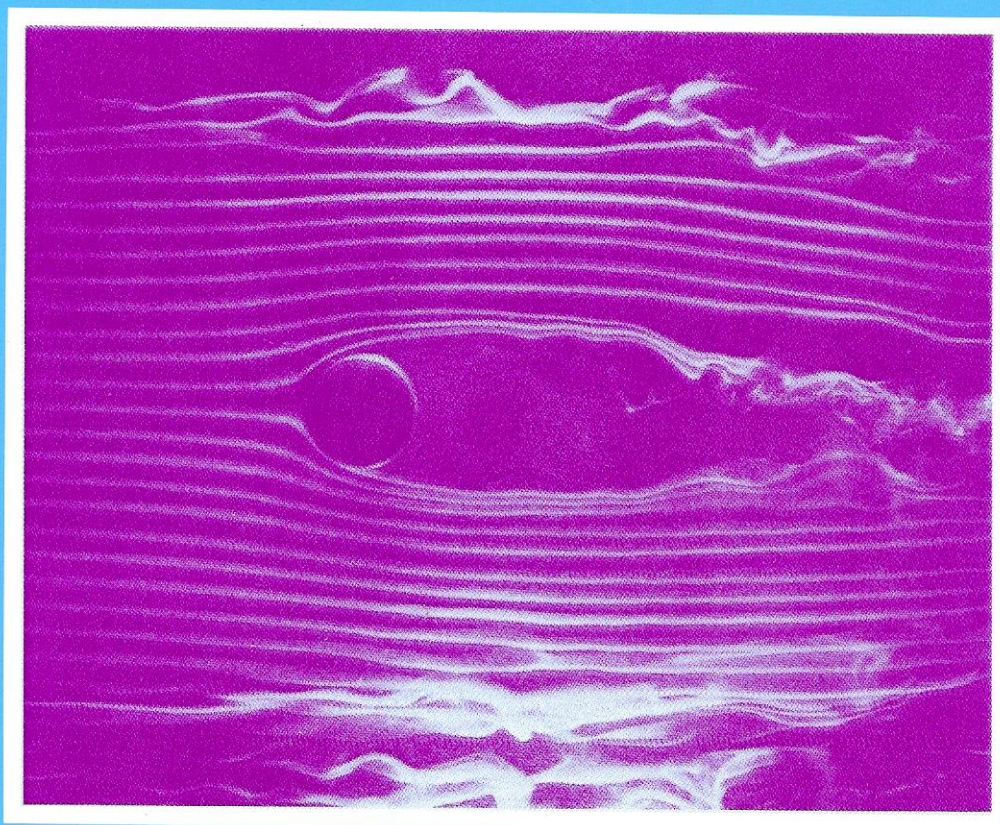
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



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Unit 8

The flow of a viscous fluid



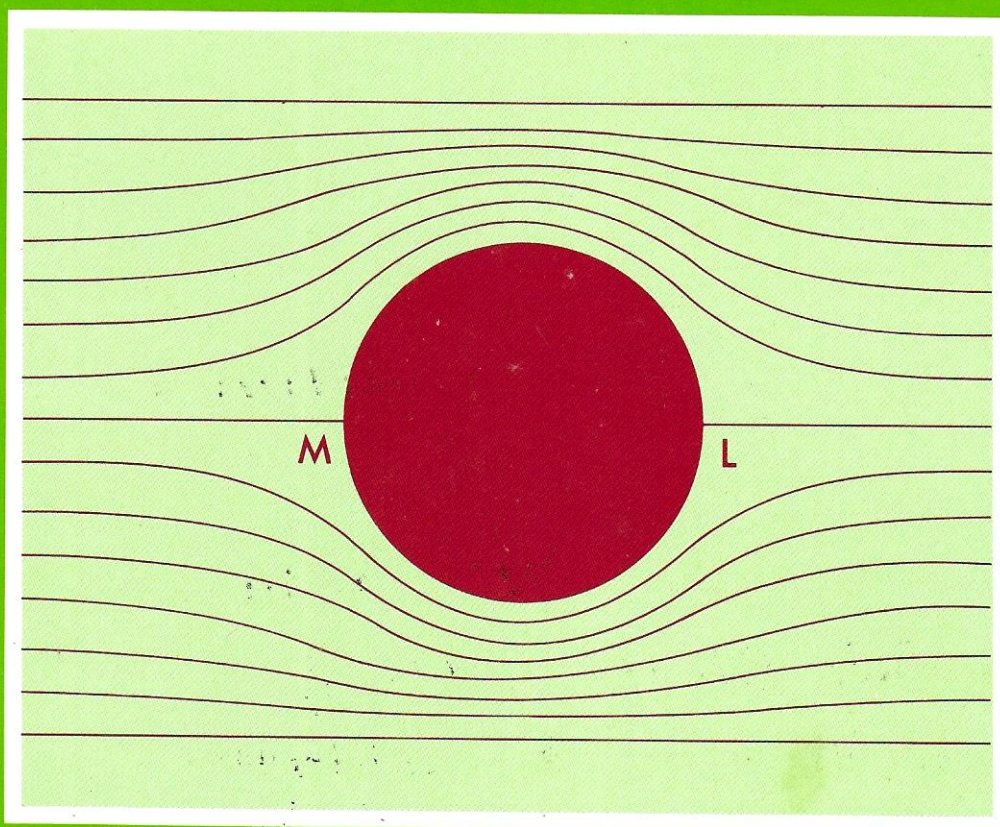
$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



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Unit 9

Second-order partial differential equations



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$

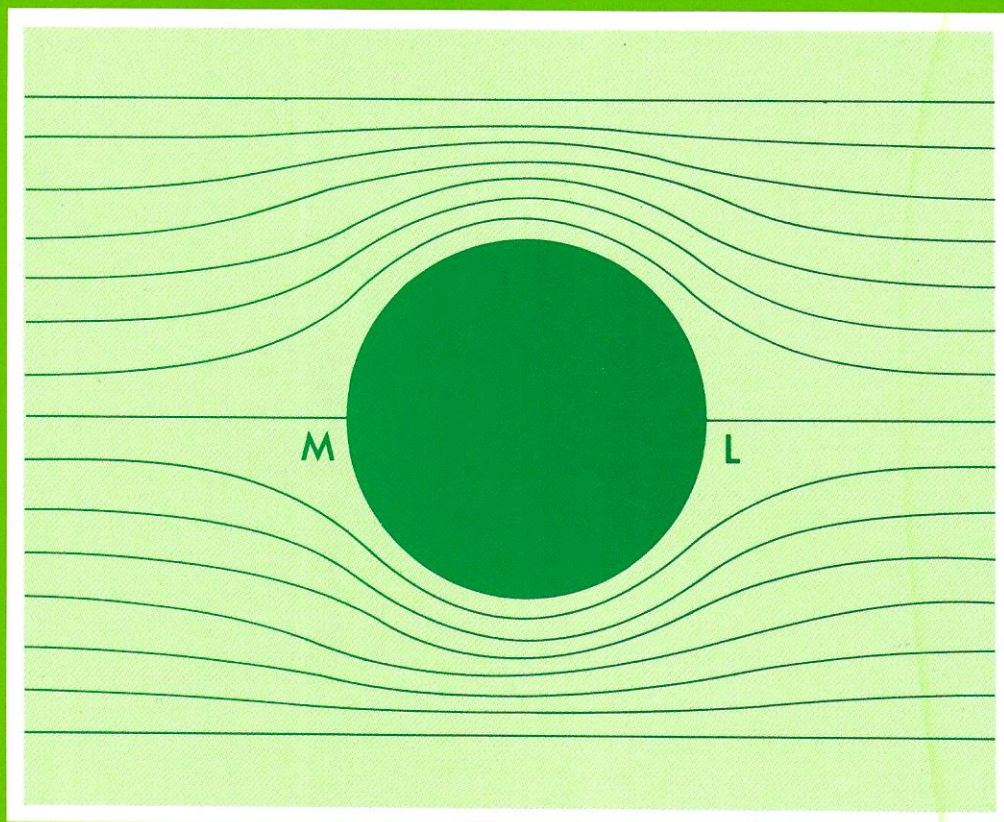
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



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Unit 10

Fourier series and power series



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$

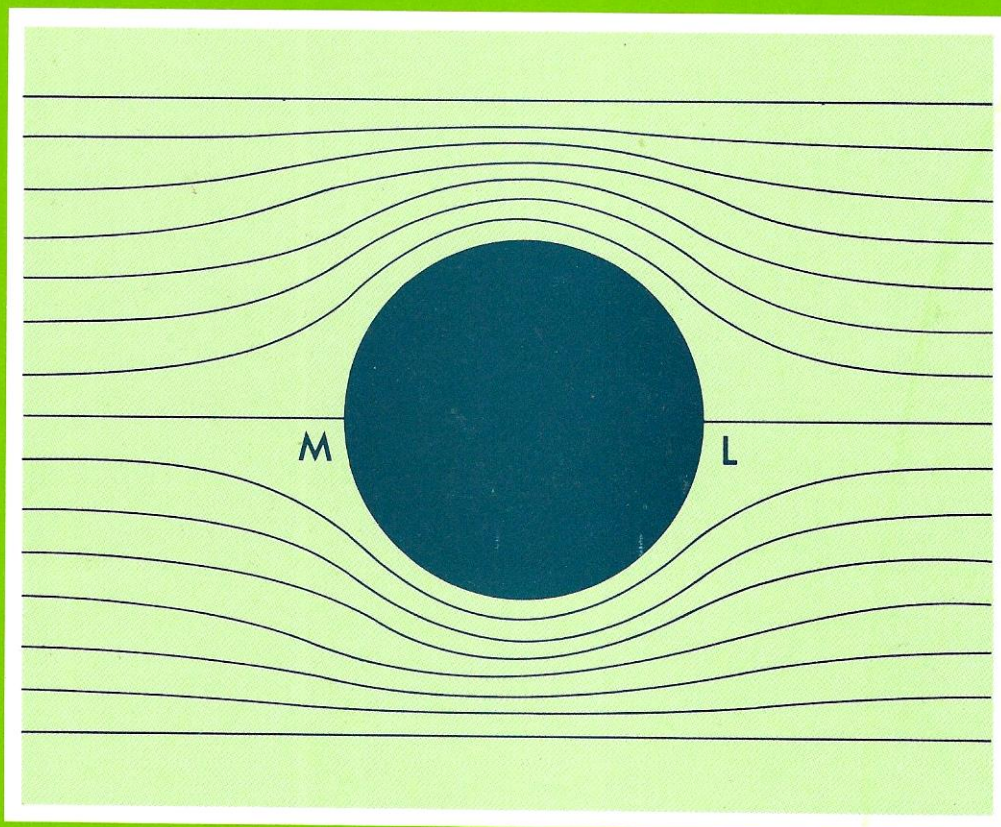
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



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Unit 11

Sturm - Liouville theory



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$

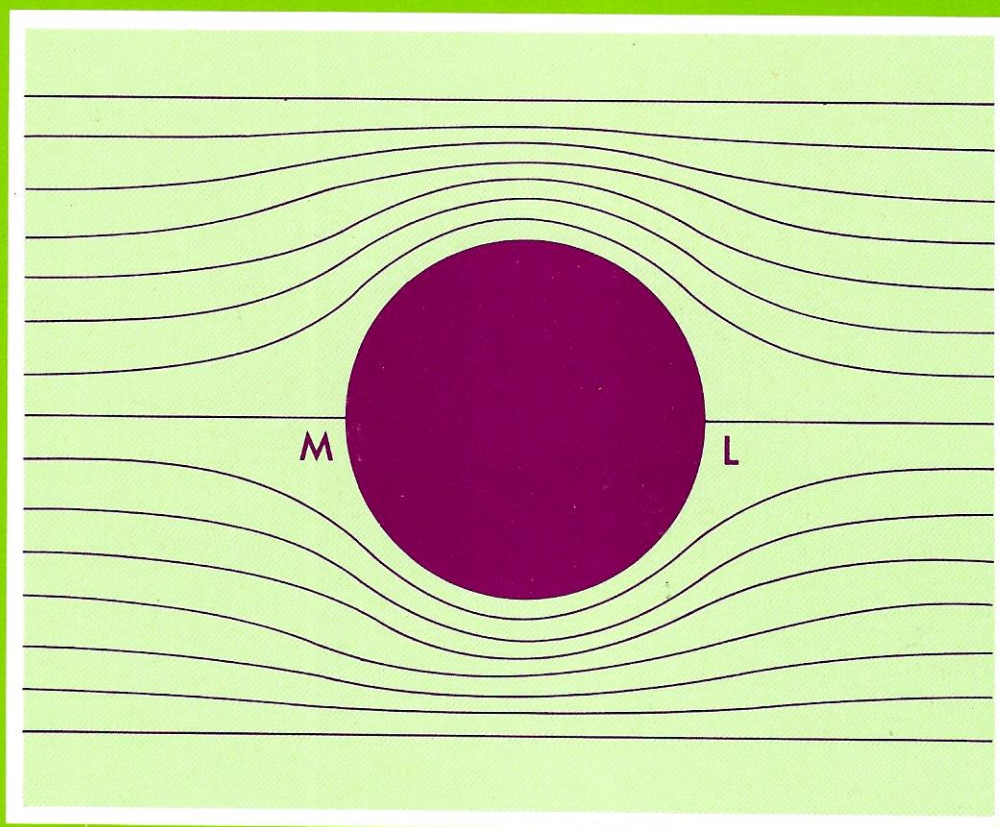
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



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Unit 12

Laplace's equation



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



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Unit 13

The wave equation



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



Mathematical Methods and Fluid Mechanics

Unit 14

Water waves



$$\rho \frac{du}{dt} = -\frac{\partial p}{\partial x} + \rho F + \mu \frac{\partial^2 u}{\partial x^2}$$
$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} = 0$$



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CONCLUSION